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THE MONIST

PLATO'S CONCEPTION OF THE COSMOS.

I.

"PYTHAGORAS was the first," says Plutarch, "who named the compass of the whole a Cosmos, because of the order which is in it."

The notion that all things knowable and all things existent form one orderly and comprehensive system, in which every event is linked with every other by causal necessity while all the elements with mechanical nicety mutually enmesh, is to us of to-day an intellectual commonplace. We make no difficulty in thinking an Everything which is made up of all things, an Entirety or a Totality which is just the commingled sum of the numberless particularities which our lives are always itemizing; and we call this Totality, this All, this Thing of things, the Universe or the World. It rarely occurs to us to question either the unity or the reality of this omnium-gatherum, which, even if it occupies a somewhat concealed position in our thoughts, is yet a well-nigh indispensable convenience; it stands an ever-ready and capacious receptacle for all the perplexities and inconsistencies which the apparent nature of things is constantly presenting, but which, we feel, are in some benign way healed by the alchemical mystery of an all-inclusive World.

Ideas are habits; and when an idea gets so fixed that the habit has become automatic, it is usually good medicine

to revive, now and again the habit-forming period, that we may judge with refreshed intelligence the safety and truth of our continued course. This is our purpose in turning to certain Greek conceptions of the world as a cosmos.

For we must remember that the notion, so familiar to us, of what they variously called τὸ πᾶν, the All, or τὸ ὅλον, the Whole, or again ὁ οὐρανός, the Heaven, or ὁ κόσμος, the Order of Things, was to the Greeks a new invention. The idea that all things are somehow one is by no means self-evident, and when it was suggested the wary Hellenic mind approached it with canny suspicion and cautious circumlocution. Is the World limited or unlimited? Is it truly One or is it Many? Does the Whole, or Totality, exhaust the All? Or indeed may not the All indefinitely transcend the Realm of Order, the Cosmos? These were questions which were raised and discussed—questions with a dangerous smack of impiety—by the men who were interested in what Xenophon characterizes as "that which is called by sophists 'the world.'"

Doubtless it was Pythagoras, as Plutarch states, or some Pythagorean, who first daringly pronounced the Whole to be a Cosmos, the realm of reality and the realm of order to be coextensive. For the Pythagoreans were the earliest of men to be entirely enamored of that first principle and foundation of law and order, the idea of number. They devoted themselves to mathematics and music and astronomy, and in the numerical analogies which they discovered in the properties of sound and in the movements of the heavenly bodies there burst upon their minds, with what must have seemed a very blaze of creative intelligence, the great conception of number in nature, which has since been the foundation of all our science. They conceived all nature to be organized according to mathematical proportions, and because they found these proportions to be most emblematically realized in musical strings

and pipes they named the principle of it a harmony, and again because they seemed to see it regnantly imaged in the motions of the heavenly spheres they regarded these too as a harmony and a music. It was indeed primarily to the heavens that the name Cosmos was given, and it was only later, when the seasons of Earth were observed to follow the periods of the Sun while the figures of the stars were regarded as prognostics of human events, that the conception of order was extended from celestial to terrestrial phenomena.

The background of Hellenic thought, like the natural thought of mankind everywhere, was pluralistic. To the normal Greek, even in the days of Plato and Aristotle, the obvious facts of life indicated not a consistent and close-locked universal scheme, but a *mêlée* of whim and purpose, blind chance and blinder fancy, while the most reasonless of all the powers he recognized was that to which he gave the name Necessity. To him it seemed evident that the affairs of men and nature are innumerable and unorganized, and while certain of the more stable aspects of existence were regarded as the charge of the Olympian gods, not even such mercurial control as emanated from the hoydenish family of Zeus divine obtained in the generality of experience: the vast majority of events were not to be explained at all; they were simply the manifestation of the hostility, indifference, idiosyncracy and anarchy which appear in the elemental facts of life.

This, I say, was the view of the normal Greek even in his classical hey-dey, as it is the view of the naive and natural man everywhere. But the foundations of our own sophisticated philosophy had been set long before, in two first conditions which, as I see it, go far to account for the whole edifice of reason.

One of these is a psychological condition. It is what is known in Kantian philosophy as the "unity of apper-

ception" and in scientific method as the "law of parcimony," or economy of thought. Essentially it is just our native simple-mindedness, expressed in the maxim, "Attend to one thing at a time." Intellectually we are unable to cope with complex facts; we have to simplify them, analyze them, in order to see them. Hence we regard simplicity as the supreme virtue, not only in reason but also in nature; and hence also our invincible conviction that reason's simplifications are more genuine than nature's empirical complexities. In spite of its multitudinous and multiplying variety the very limitations of our intellectual powers compel us to see Nature as one, as a unity, and thus out of chaos is created an orderly world.

Such is the inner condition, but it is mightily helped outwardly by the natural allegory of Sky and Earth, Day and Night, Summer and Winter. These antitheticals seem to form a great division of Nature into the Intelligible and the Unintelligible: Sky and Day and Summer not only symbolize but embody motion and light and life, which are in turn the image and essence of reason; while Earth and Night and Winter no less surely body forth the inert and void and deathly realm of anti-reason. Thus we have a realm of order, Cosmos, set over against a realm of disorder, a Chaos; and because the orderly Sky images the rulership of reason, and because Day is the revealer and Summer the life-giver, these powers are regarded as friendly to man and in the great contention of Nature as encroaching upon and subduing the dark forces of Chaos.

Such a sense of duality is omnipresent in human thought. Its metaphors are the very breath of life of poetry, and even in philosophies which deny its reality the problems to which it gives rise—problems of the formal and material, spiritual and physical, good and evil,—are the crucial perplexities. Greek thought is no exception to the rule. Already in the epic theogonies Uranus and Gaea,

Sky and Earth, appear as ancestral and gigantic forms of creation emerging from primeval chaos....

"First Chaos was, and then broad-bosomed Earth....
And Earth bare starry Heaven, thence to be
The habitation of the blessed gods."

This is the Hesiodic genesis, and the Orphic differs from it only in making Heaven and Earth a coequal and wedded pair, from whose union multitudinous nature was begotten. Euripides preserves it in the utterance of the seeress Melanippe:

"It is not my word, but my mother's word,
How Heaven and Earth were once one form; but stirred,
And strove, and dwelt asunder far away:
And then, re-wedding, bore unto the day
And light of life all things that are, the trees,
Flowers, birds and beasts and them that breathe the seas,
And mortal man, each in his kind and law."[1]

This dualism of the epic age passed over into the philosophic tradition with little more than a change of names. In place of Heaven and Earth, the antithesis is set between Chaos and Nous, Anarchy and Intelligence, or between Chaos and Cosmos, Void and Order,—though we must remember that the word οὐρανός persisted as a synonym of κόσμος even with Plato and Aristotle, and that κόσμος itself was at first used of the heavenly firmament, and only with advancing insight into the orderliness of the world beneath the spheres was it made to include terrene nature.

The lesson of intelligence was in fact learned first of all from observation of the heavens. No phenomena so vividly impress the natural mind with a sense of their divinity as do the regular and brilliant courses of the heavenly bodies. Repetition is the gateway and light is the outer image of learning, and in the sun and moon and stars we have our permanent exemplars of repetition and light.

"All mankind thou guidest as a single being;
Expectantly, with raised head, they look up to thee!"

[1] Gilbert Murray's translation.

says a Babylonian hymn to the sun, for which the nineteenth psalm—

> "The Heavens declare the glory of God,
> And the firmament sheweth his handywork"—

is only a later parallel. Plato, in describing the works of the Demiurge, tells how "of the heavenly and divine, he created the greater part out of fire, that they might be the brightest of all things and fairest to behold, and he fashioned them after the likeness of the universe in the figure of a circle, and made them follow the intelligent motion of the supreme, distributing them over the whole circumference of heaven, which was to be a true cosmos or glorious world spangled with them all over."[2] And in another passage Plato derives from the image of the heavens, as does the psalmist, his conviction of the goodness of God: for if, he says, "we say that the whole path and movement of heaven, and of all that is therein, is by nature akin to the movement and revolution and calculation of mind, and proceeds by kindred laws, then, as is plain, we must say that the best soul takes care of the world and guides it along the good path." Perhaps the sublimest expression of this thought in Greek literature is Aristotle's characterization of Xenophanes: "He cast his eyes upon the expanse of Heaven, and saw that it was one, and that one God."

Thus the heavens were at once the embodiment of reason and divinity, the symbol of divine rulership and the exemplar of divine perfection. But it was the reverse of obvious that either the mathematical regularity of the heavenly reason or the perfection of heavenly form extend to the world beneath the moon. What seems to have been really the first suggestion that such is the case was the Pythagorean discovery that musical intervals vary with the length of the sound-producing strings according to certain simple and regular numerical ratios. This discov-

[2] This and other citations from Plato are in Jowett's translation.

ery burst upon men's minds as a sudden revelation of order where order had hitherto never been suspected, and in their first delirious application of it the Pythagoreans seemed to see number everywhere, in the world of change below as in the world of constancy above, in the conduct of men as in the conduct of gods and stars, and so they proclaimed the Whole to be a One, whose emanating numbers gave coherence and system to all things, and they named this systemic All a Cosmos.

There remained one further step. Xenophanes had seen God in the heavens; Pythagoras had lifted Earth up into the Cosmos; but neither had as yet perceived that the world of sense and of physical numbers is only a symbol and an image of the true realm of law, that the cosmic citadel must be sought inwardly in thought and not outwardly in fact. This had been darkly intimated by the dark Heraclitus. "Better is the hidden harmony than the manifest," he had said; and again, "In one thing is wisdom, to know the reason by which all through all is guided." But it was Socrates who first clearly and explicitly emphasized the inner nature of the cosmic principle. "Socrates was the first," says Cicero, "to call philosophy down from the sky, and to settle it in the city and even introduce it within the house, and compel it to inquire concerning life and death and things good and ill." Probably, in saying this, Cicero, like Xenophon, merely saw Socrates turning from astronomy as from a vain speculation. The truth of Socrates' mission is perhaps better indicated by Aristotle's statement that it was Socrates who invented definition. We know what he strove to define—courage and temperance and justice and wisdom, the principles of conduct and the laws of an orderly life. Socrates was seeking cosmos, reason, not in the physical image, but in the spiritual reality. That Socrates was genuinely interested in physical science there is every reason to believe,

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but his final attitude is best expressed in the words which Plato puts into his mouth, "Those who elevate astronomy into philosophy appear to me to make us look downward and not upward."

The predecessors of Plato had modelled two great conceptions. The physical and mathematical thinkers had evolved the grandiose notion of a Cosmos, an Order, written upon the face of Chaos. Heraclitus and, far more distinctly, Socrates had proclaimed this order of nature to be only the outward image and reflection of the inner order of reason. Pythagoras and Heraclitus and Socrates, more than all others, were the teachers of Plato, and it was from the inspirations of their insights that he drew his own magnificent vision of the world.

II.

The vivid impression one derives from a reading of Plato is of the intensity of his conviction of the unreality of sensible things. The world of sense, of sight and hearing and taste and touch, in which most men chiefly dwell is for him a shadow world. At its best it is but a symbol obscurely imitating the character of the reality which it veils; in its normal function it is a delusional mirage; and at its worst, when it conveys the deception of knowledge, it is the fount of corruption and the seed of damnation. The Greek argument against our commonsense conviction that what we see and touch is real is about as follows: All objects of sense suffer perpetual change; they never *are* this or that, but are always in a process of becoming or of ceasing to be this or that; hence, we cannot justly describe them as being anything, or indeed as having any true existence of any sort. Heraclitus remarked that one cannot bathe in the same river twice, and Cratylus, the sceptic, after remarking that we cannot in fact bathe in the same river even once, finally, as Aristotle tells us,

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ceased speech altogether on the ground that it was impossible to say any thing that is true; to inquisitors he would reply merely by a wagging of the finger, his mutely eloquent asseveration of his master's dogma that "All things flow." Plato accepted this doctrine, as he also accepted Socrates's conception that ignorance is essential vice, and combining the two, to the sceptical he added a moral condemnation of the world of sense: not only does it not give us truth, but because, as he says, "ignorance is the aberration of a mind bent on truth," through the intensity of its illusions it betrays the soul's integrity.

The Cratylean denial of the possibility of discourse is thus, for Plato, the proclamation of moral ruin, and at such his sanity revolts. Nor is the way of salvation hard to find. If sense be false, ideas may yet be true, and in its own proper world discourse may be dealing with reality. "Knowledge"—these are Plato's words—"does not consist in impressions of sense, but in reasoning about them; in that only, and not in the mere impression, truth and being can be obtained." And again: "Things of which there is no rational account are not knowable....things which have a reason or explanation are knowable." Plato's "world of Ideas," as it is called, is in fact but the assertion that our speech is significant, and that this significance, not the courses of sense, is what we mean by reality. "The word expresses more than the fact" and "in the nature of things the actual must always fall short of the truth."

Plato's idealism is thus simply a sane and unconquerable conviction that there is a realm of truth, and his whole philosophy is an effort to find out this truth. In the *Phaedrus* he speaks of truth as "the pilot of the soul"; in the *Philebus* he asserts that the soul has "a power or faculty of loving truth and of doing all things for the sake of it"; and in the *Phaedo* he makes Socrates, about to take the hemlock, preface his great argument for the soul's immor-

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tality with a wise caution against the bias of desire, "I would ask you to be thinking of the truth and not of Socrates."

Yet Plato has no illusory notion that truth is of easy access. Immersed as we are in a sea of distorting sensation, our knowledge at its best is only a faith. "For there is no light of justice or temperance or any of the higher ideas which are precious to souls in the earthly copies of them: they are seen through a glass darkly." In the famous image of the den, wherein mankind are the chained prisoners, with their eyes fixed upon the shadows of reality, Plato reminds us that even were our eyes opened to the upper world the light of reality would sear our vision. All that we can hope for is such intimations of the truth as we can gather from the allegory of nature.

And with a curious astuteness he emphasizes the affinity of vision—"the clearest aperture of sense"— to the inner perception of truth. "Sight in my opinion," says Timaeus, "is the source of the greatest benefit to us, for had we never seen the stars and the sun and the heavens, none of the words which we have spoken about the universe would ever have been uttered. But now the sight of day and night, and the months and the revolutions of the years, have created number, and have given us a conception of time, and the power of inquiring about the nature of the universe; and from this source we have derived philosophy, than which no greater good ever was or will be given by the gods to mortal men.... God invented and gave us sight to the end that we might behold the courses of intelligence in the heavens, and apply them to the courses of our own intelligence which are akin to them, the unperturbed to the perturbed; and that we, learning them and partaking of the natural truth of reason, might imitate the absolutely unerring courses of God and regulate our own vagaries."

In this remarkable passage Plato compresses not only

the actual history of science, but its psychological foundations and its metaphysical ends, with a precision truly astonishing. I cannot dwell upon the multitude of analogies that it suggests, but the fundamentals are obvious; for the sense of sight is in fact the pattern of intelligence; perception of the heavens has given us our measures of time, and has created number and the science of the calendar which is the parent of all the sciences and of philosophy as well; and again the constancies of the celestial bodies have ever seemed to men, as Plato says, the regulation and the healing of their own errant ways. The whole life of reason is summarized and prophesied in this natural allegory.

And yet, let us repeat, it remains for Plato throughout an allegory. All science is an allegory and an art. What men call nature, the experiences that in human life stand over against our essential humanity, is after all unreal. It may image reality because it is the product of creative reason, but beyond this power of imaging its only being is scenic and mirage-like.

"The starry heaven which we behold is wrought upon a visible ground, and therefore, although the fairest and most perfect of visible things must necessarily be deemed inferior far to the true motions of absolute swiftness and absolute slowness, which are relative to each other, and carry with them that which is contained in them, in the true number and in every figure. Now, these are to be apprehended by reason and intelligence, but not by sight... The spangled heavens should be used as a pattern and with a view to that higher knowledge; their beauty is like the beauty of figures or pictures excellently wrought by the hand of Daedalus, or some other great artist, which we may chance to behold; any geometrician who saw them would appreciate the exquisiteness of their workmanship, but he would never dream of thinking that in them he

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could find the true equal or the true double, or the truth of any other proportion....And will not the true astronomer have the same feeling when he looks at the movements of the stars? Will he not think that heaven and the things in heaven are framed by the Creator of them in the most perfect manner? But he will never imagine that the proportions of night and day, or of both to the month, or of the month to the year, or of the stars to these and to one another, and any other things that are material and visible can also be eternal and subject to no deviation—that would be absurd."

Where the ancients said "astronomy" we say "physics," remarks a savant of our own day; and is it not obvious that Plato's words hold with perfect truth of our own science? For we, like Plato, to not look to the visible and sensible world for our realities, but to an ideal world which is only faintly intimated by the riddle of the senses. Whether it be as in our mechanical sciences a world of atoms and molecules or of ether vortices or of electrons and ions, or as in our biological sciences a world of genera and species, in every case we hypothecate a realm of forms, of ideas, as the essential reality of all natural phenomena. We vary no whit from Plato in all this; and indeed, little as they may suspect it, all our scientists are good Platonians.

But where we do vary from Plato is in the kind of value which we set upon our ideas. For we regard our scientific knowledge as ultimate and as a kind of divine possession in itself, whereas Plato held it to be only a means whereby men can dimly approach the being of divinity. In his own phrase we are "thrice removed from the king and the truth": behind the world of sense is the world of mathematical forms which are in turn but the reflection of the divine intelligence. Sense is the allegory of science, but science itself is only our human parable of divinity—a myth whose meaning is the mind of God. Science is thus

a purely human instrument, and truth, our human, intellectual truth, is but the device whereby we adumbrate the nature of being. "The Deity," says Plutarch in one of his expositions of Plato, "stands in no need of science, as an instrument to withdraw his intellect from things engendered and to turn it to the realities; for these are all in him, and with him, and about him." It is only the weakness of human insight that makes the world-myth a significant myth.

III.

Plato, his critics are accustomed to say, resorts to allegory, to what he himself calls myth, when he encounters problems with which rational analysis alone is unable to cope. The lordly tales which adorn his dialogues these critics view as imaginative ornaments which Plato himself takes only half seriously. This I believe to be a misunderstanding. It is characteristic of these myths that they are introduced not when Plato is analyzing the nature of being, but when he has passed to a discussion of becoming, that is, when cosmic history rather than metaphysical organization is his theme. Now it is this province of becoming, which we should call the field of empirical science, which is, in Plato's view, itself an allegorical reality. And in resorting to allegory for its description he is but emphasizing the duplex nature of the fact. There is no field of discourse where positive statement is so easy and so dangerous as in the field of science (in our modern sense), and in discussing the problems of change Plato employs myths primarily in order that he may avoid dogmatism. Empirical science is for him a work of human art, just as the empirical universe is God's work of art; and he would not have us forget, what we are so prone to forget, that our constructions of cosmic realities give us at best but a verisimilitude, or as he would say, an "imitation" of the truth. In speak-

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ing of the empiric world, he repeats again and again, we can use but the "language of probability," and the language of probability is myth.

When therefore Plato, in the language of probability or of myth, sketches for us the cosmic drama which is the history of the world, it is with no Laplacean confidence in the invulnerability of his representation. Rather he is aware that at the core it cannot be the essential truth of the cosmos: science is given us in order that we may "imitate," as he says, "the absolutely unerring courses of God and regulate our own vagaries": it is not and cannot give dogmatic knowledge. "Law and order," to quote once more, "deliver the soul"; and there is a trenchant difference between this and our modern conception that the soul is but an automatic reflection of external laws and orders.

The motive which animates Plato's cosmological speculations is thus clearly a humanistic motive. He is concerned for truth, but only for such truth as bears directly upon men's conduct, and this he does not expect to find in the sensible world. For him, as for Dante, the world in time and space is but the vesture of man's life, whose essence and reality is to be sought in that divine nature of which apparent nature is the image. Truth, then, must be appraised, and the appraiser is the Good and the Perfect,—for "nothing imperfect is the measure of anything."

The conception of a cosmic drama—a world-play having, as Aristotle would say, a beginning, a middle and an end, a complication and a solution,—is not new with Plato. It appears in the theogonic epics and in the notions of the physical philosophers of the earlier period. But it is with Plato that the proper motive of the plot appears; and this is the striving for the good. With Plato's predecessors the moral problem had been (as it is to our scientists) adventitious; with Plato it is central, and we can understand his science of first and last things only when we see in it,

as he saw in nature, a cosmic staging of the search for salvation.

Genesis and eschatology represent respectively the complication and solution of the plot. Genesis, the tale of origins, is treated most completely in the *Timaeus*; cosmic justice and its judgments is the theme of the speculative cosmology of Socrates in the *Phaedo* and of the vision of Er in the *Republic*. In these and in allied passages Plato draws for us his world emblem.

Plato begins his genesis, in the *Timaeus*, with an assertion of dualism. "First," says Timaeus, "we must make a distinction and ask, What is that which always is and has no becoming; and what is that which is always becoming and never is? That which is apprehended by intelligence and reason is always in the same state; but that which is conceived by opinion with the help of sensation and without reason, is always in a process of becoming and perishing and never really is." In its inception this dualism is a logical one, hypostatized into the familiar Platonic antithesis of the World of Sense and the World of Ideas. But very speedily we perceive that the moral antithesis of good and evil is in it also. The kernel of Plato's thought is the old philosophical dualism of Nous and Chaos, and even the older mythic dualism of Heaven and Earth; and, as does the earlier thought, he identifies Mind and Light with Goodness, and Disorder and Darkness with Evil.

"God desired that all things should be good and nothing bad, so far as this was attainable. Wherefore also finding the whole visible sphere not at rest, but moving in an irregular and disorderly fashion, out of disorder he brought order, considering that this was in every way better than the other. Now the deeds of the best could never be or have been other than the fairest; and the creator, reflecting on the things which are by nature visible, found that no

unintelligent creature taken as a whole was fairer than the intelligent taken as a whole; and that intelligence could not be present in anything which was devoid of soul. For which reason, when he was framing the universe, he put intelligence in soul, and soul in body, that he might be the creator of a work which was by nature fairest and best. Wherefore, using the language of probability, we may say that the world became a living creature truly endowed with soul and intelligence by the providence of God."

In these words of Timaeus, Plato outlines his conception of creation. God, perceiving the disorder of Chaos, designs to redeem it by imparting to it the image of mind, of Cosmos, order. He creates it, therefore, in the likeness of a perfect animal (παντελές ζῷον), "the very image of that whole of which all other animals both individually and in their tribes are portions." First he created its soul, the *anima mundi*, "to be the ruler and mistress, of whom the body was to be the subject," organized from the categories of thought, from identity and difference and essence, in harmony of number. Afterwards he gave it body, interfusing with the visible body the rational soul, so that the whole universe of being became one animal endowed with soul (ζῷον ἔμψυχον).

"And he gave to the world the figure which was suitable and also natural. Now to the animal which was to comprehend all animals, that figure was suitable which comprehends within itself all other figures. Wherefore he made the world in the form of a globe, round as from a lathe, having its extremes in every direction equidistant from the center, the most perfect and the most like itself of all figures; for he considered that the like is infinitely fairer than the unlike. This he finished off, making the surface smooth all round for many reasons; in the first place because the living being had no need of eyes when there was nothing remaining outside of him to be seen; nor of ears

when there was nothing to be heard; and there was no surrounding atmosphere to be breathed; nor would there have been any use of organs by the help of which he might receive his food or get rid of what he had already digested, since there was nothing that went from him or came into him: for there was nothing beside him. . . . And, as he had no need to take anything or defend himself against any one, the Creator did not think it necessary to bestow upon him hands: nor had he any need of feet nor of the whole apparatus of walking; but the movement suited to his spherical form was assigned to him, ... and he made the universe a circle moving within a circle, one and solitary, yet by reason of its excellence able to converse with itself, and needing no other friendship or acquaintance. Having these purposes in view he created the world a blessed god."

"When the father and creator saw the creature which he had made moving and living, the created image of the eternal gods, he rejoiced, and in his joy determined to make the copy still more like the original; and as this was eternal, he sought to make the universe eternal, so far as might be. Now the nature of the ideal being was everlasting, but to bestow this attribute in its fullness upon a creature was impossible. Wherefore he resolved to have a moving image of eternity, and when he set in order the heaven, he made this image eternal but moving according to number, while eternity itself rests in unity; and this image we call time." Time came into being with the heavens which measure it, and will be dissolved with them, says Plato; but space is of another origin. For besides the reason which gives cosmic form there is another cause of being, a principle of limitation which Plato calls necessity. We must conceive, he says, of three natures: first, that which is in process of generation, and this would be the world of nature as we experience it; second, that in which the generation takes place, and this is the recipient or

matrix of nature; and third, that of which the generated world is an image, and this is the cosmic reason or form. "We may liken the receiving principle to a mother, and the source or spring to a father, and the intermediate nature to a child," he says, and we think immediately of the mythopoetic union of Earth and Heaven and of the Life of Nature which is its offspring. But for Plato this is a mere trope; he does not rest without being scientifically explicit. There are three kinds of being: that which is uncreated and indestructible, changeless, eternal, imperceptible to any sense, open only to the contemplation of the intelligence, and this is the principle of the Father, the ideal or formal essence of the world; again, that which is sensible and created and always in motion, the Child, the world of change and life; and finally, there is a third nature, the Mother, which, like the Father, is eternal and admits not of destruction, which provides a home for all created things, and is apprehended "without the help of sense, by a kind of spurious reason, and is indeed hardly real." This nature is space, and we "beholding as in a dream, say of all existence that it must of necessity be in some place and occupy a space, but that what is neither in heaven nor in earth has no existence."

This mothering space which is hardly real, yet is the cause of the determinism of nature, Plato identifies as the material element of being. As pure matter it is purely indeterminate, but it is receptive of all determinations. The four elements, earth, air, fire and water, are formed from it, for "the mother substance becomes earth and air, in so far as she receives the impressions of them." Plato's conception of the formation of these elements from the original substance was as purely mathematical as are our modern physical notions. "God fashioned them by form and number," he says; and the forms which he assigned were the forms of the regular solids. Thus the form of the fiery

element is the pyramid, of air the octahedron, of water the icosahedron, of earth the cube. The fifth solid, the dodecahedron is the form of the universe as a whole, or perhaps one might say the scaffold upon which the spherical universe is constructed. Further, these elements are themselves compounded of simpler mathematical forms, the pyramid, octahedron and icosahedron of scalene, the cube of equilateral triangles; so that if we regard the elements as molecules, we may view the triangles as atoms of the material substrate.

Doubtless it was this geometrical account of matter which gave rise to the saying ascribed to Plato that "God always geometrizes,"—for God, says Plutarch in his commentary on the saying, made the world in no other way than by setting terms to infinite and chaotic matter. But it is not with the mathematical aspect of Plato's theory that we are here most concerned, but with its moral bearings. For it is in matter that Plato finds the root of evil, and, if we may so put it, the villainy of the world. In framing the inhabitants of the world, according to the account of Timaeus, the Creator made first the race of gods, perfect and immortal; but of the race of men he made only the souls, their bodies were handed over to the created gods to be composed of perishable matter. "The part of them worthy of the name immortal, which is called divine and is the guiding principle of those who are willing to follow justice and you (the gods)—of that divine part I will myself," saith the Creator, "sow the seed, and having made a beginning, I will hand the work over to you. And do ye then interweave the mortal with the immortal, and make and beget living creatures, and give them food, and make them to grow, and receive them again in death."

And having made souls equal in number to the stars, and having assigned each soul to a star, and there placed

them as in a chariot, God "showed them the nature of the universe, and declared to them the laws of destiny, according to which their first birth would be one and the same for all,—no one should suffer a disadvantage at his hands,"—and he showed them how "he who lived well during his appointed time was to return and dwell in his native star, and there would have a blessed and congenial existence; but if he failed in attaining this," he would be reborn into some brute who resembled him in evil nature, nor would his toils and transformations cease until the principle of reason had enabled him to overcome "the turbulent and irrational mob of later accretions, made up of fire and air and water and earth" and return to his first and purer state. And "having given all these laws to his creatures, that he might be guiltless of future evil in any of them, the Creator sowed some of them in the earth, and some in the moon, and some in the other instruments of time; and when he had sown them he committed to the younger gods the fashioning of their mortal bodies, and desired them to furnish what was still lacking to the human soul, and to rule over them, and to pilot the mortal animal in the best and wisest manner which they could, and avert from him all but self-inflicted evils."

In these passages we see the *rationale* of the Platonic doctrines of anamnesis and metempsychosis, or recollection and transmigration. The great image in the *Phaedrus* of the soul in its chariot driving the unruly and the ruly steed, and the descriptions of a future-world judgment in the *Phaedo* and *Republic,* in which these doctrines are presented, appear as necessary scenes in the cosmic drama. The motive of that drama is the conflict of form and matter, Nous and Chaos, which on its theological side is the conflict of God and Necessity as the two principles of being, and in its moral aspect is the strife of Good and Evil. In each of these senses Plato is a dualist; and if he describes chaos

and matter and evil in negative terms, this is not because he views them as non-existent (as our modern idealists seem to do), but because he regards them as impermanent, and hence as unreal; for Plato defines the real as the permanent, never, however, meaning thereby to deny genuineness of our experience of change and hence of imperfection and evil.

Nevertheless, Good and Evil, God and the Devil, are not in Plato's conception coordinate powers. Their difference is a difference of dramatic position. In the world-conflict we, as human beings, are all enlisted on the side of the good, and if we are traitorous to it this is because of the deceit of the enemy. "For as we acknowledge the world to be full of many goods and also of evils, and of more evils than goods, there is, as we affirm, an immortal conflict going on among us, which requires marvelous watchfulness, and in that conflict the gods and demigods are our allies and we are their property." No Persian has ever stated this fundamental dualism more emphatically nor adhered to it more uncompromisingly. From it Plato deduces the ascetic rule of life which recurs in his writings so repeatedly. "Evils," says Socrates in the *Theaetetus*, "can never pass away; for there must always remain something which is antagonistic to good. Having no place among the gods in heaven, of necessity they hover around the mortal nature and this earthly sphere. Wherefore we ought to fly away from earth to heaven as quickly as we can." And from it, too, comes Plato's clear-eyed perception that the idea of good holds the hegemony over all our interests, scientific and esthetic as well as moral. It is the good—as our pragmatists say—which makes truth true and is indeed the measure of reality. For "that which imparts truth to the known and the power of knowing to the knower is what I would have you term the idea of good, and this you will deem to be the cause of science, and of

truth in so far as the latter becomes the subject of knowledge; beautiful too, as are both truth and knowledge, you will be right in esteeming this other nature as more beautiful than either; and as light and sight may be truly said to be like the sun and yet not to be the sun, so in this other sphere, science and truth may be deemed to be like the good, but not the good; the good has a place of honor yet higher."

IV.

Let me briefly recapitulate Plato's view. In the beginning were God and Chaos. And God strove to impress the spirit of order, which is his own divine spirit, upon the face of the Void. And in his own image he created a soul of the World, and the name of this soul is Cosmos, Order. And to this divine soul he united a body, hewn from Chaos, and this soul in this body forms the visible Heaven and all that is therein. And he created inhabitants for the world which he had made, the race of gods and of demigods and the race of mortal men; and these were to be his allies and his helpmates in the redemption of Chaos. For Chaos is ruled by blind Necessity, and the horror of its blindness enters into all being in which it has a share; so that not men nor demigods nor gods are free from the peril of Darkness, which is the peril of their material and temporal being. Wherefore it behooves them, men and gods, to strive nobly after the Good, holding fast to the image of divinity which is in them. And to this strife there is and there can be no end. For Chaos is coequal with God, infinite in change as God is infinite in might; and the conflict of the two is the eternal struggle for the world's salvation which is the world's life.

In conclusion, I would say a word in regard to the wonderful vitality of Plato's thought; for to no other philosopher has it been given to lay such lasting hold at once upon men's reason and upon their affectionate imagination. I think the clue to this will appear if we compare his atti-

tude with that of his great pupil and competitor toward the man from whom both derive their inspiration. For Aristotle, the arch-intellectualist, saw in Socrates but the inventor of definition—"two things may be fairly ascribed to Socrates, inductive arguments and universal definition"—and he made definition the very core of his own metaphysics. But for Plato Socrates is first and last that "midwife of souls" which he would have himself to be. Plato, in other words, had caught what Aristotle missed, the central spirituality of Socrates's teaching.

Plato is a great dialectician and a master of the things of the intellect, but he knew, as Socrates had taught, that reason alone cannot bring us to the truth, and that science is no capable vessel of reality. When "all philosophers proclaim, as with one voice, that mind is the king of heaven and earth—in reality they are but magnifying themselves," he says; for he knows well that beyond the symbols of sense, which are the symbols of our reason, there is a more splendid reality. We can see this other-world truth but as in a glass darkly; we can speak of it only in myth and allegory; we can hope for its realization never save in those aeon-parted moments of the cosmic cycles when the soul, after its hour of agony, has brought its steeds to that outer revolving heaven whence the things that are beyond stand revealed. And "of that heaven which is above the heavens, what earthly poet ever did or ever will sing worthily?.... There abides the very being with which true knowledge is concerned; the colorless, formless, intangible essence, visible only to mind, who is the pilot of the soul. The divine intelligence, being nurtured upon mind and pure knowledge, and the intelligence of every soul which is capable of receiving the food proper to it, rejoices at beholding reality, and once more gazing upon truth, is replenished and made glad, until the revolution of the worlds brings her round again to the same place."

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Such is the beatific vision, and "how can he who has magnificence of mind and is the spectator of all time and all existence think much of human life?" Surely he will value it only for this spiritual prospect which it promises; "he will look at the city which is within him" whereof the pattern is the heavenly city; and "he will live after the manner of that city, having nothing to do with any other."

Is it not because of this faith in the spiritual reality of the world-life, which is a faith in the spiritual power of mankind, that Plato has brought conviction to the minds of his fellows, generation after generation, the edifice of his thought standing secure amid the rise and decay of competing systems? And is there other measure of truth than this?

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BERKELEY'S LOGIC OF MATHEMATICS.*

THAT Berkeley was keenly interested in mathematics is well known. In the *Commonplace Book* a great deal of attention is paid to mathematical questions; it is noticeable, indeed, that in its pages Berkeley refers to mathematicians far more frequently than to philosophers. The extent of his interest in mathematics is indicated also by a group of early writings, *Arithmetica absque Algebra aut Euclide demonstrata,* and *Miscellanea Mathematica* which includes papers "de Radicibus Surdis," "de Cono Aequilatero et Cylindro eidem Sphaerae circumscriptis," "de Ludo Algebraico," and "Paraenetica quaedam ad studium matheseos praesertim Algebrae." Both of these tracts were written in 1705 and first published in 1707. Belonging to the same period is the essay "Of Infinites," which is in part concerned with the infinitesimal calculus. Berkeley deals with mathematical questions also in *The Principles* (1710) and in *De Motu* (1721), and his criti-

* The following article contains, in its treatment of Berkeley's early work which was not published for generations after it was written, a new and important contribution to the history of mathematics. It will also be of interest to our readers to know that editions of the books by Barrow and Wallis mentioned in this article are in preparation. They are edited by Mr. J. M. Child and will appear in the "Open Court Classics of Science and Philosophy." Further, in the same series a small volume by Prof. Florian Cajori on the history of fluxional concepts from the time of Newton is also in preparation. It will contain a detailed account of the *Analyst* controversy. Finally it is to be noticed that Berkeley's doctrine of "compensation of errors" in the calculus was later advocated by the eminent mathematicians Lagrange and Lazare Carnot.—Proofs of this article did not reach the author who was absent on military service.—Ed.

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cisms of the logical basis of the infinitesimal calculus in *The Analyst* (1734) and *A Defence of Free-Thinking in Mathematics* (1735) are of considerable importance in the history of mathematics.

In this paper I propose to consider the mathematical views stated in Berkeley's *Commonplace Book* and *Analyst*. In both cases he is concerned mainly with the logical basis of mathematics.

Berkeley very clearly perceived that his "new principle" involved difficulties with regard to the nature of mathematics. The "new principle" implied that lines consist of a finite number of points, that surfaces consist of a finite number of lines, and that solids consist of a finite number of surfaces. Thus ultimately all geometrical figures are composed of complexes of points, which are regarded by Berkeley as ultimate individualities. These indivisibles are *minima sensibilia,* the minutest possible objects of sense. It is impossible that the *minimum sensibile* should be divisible, because in that case we should have something of which our senses could not make us aware, and that, Berkeley believes, is simply a contradiction.[1]

Sensation, then, is the test of all geometrical relations. Thus equality depends simply on our inability to distinguish in sense-perception. "I can mean nothing by equal lines but lines which it is indifferent whether of them I take, lines in which I observe by my senses no difference."[2] He explicitly considers the claims of imagination and pure intellect to judge of geometrical relations, and summarily rejects their pretensions. Imagination, he points out, is based on sensation, and has no other authority than that of the senses. It has no means of judging but what it derives from the senses, and, as it is removed by one stage from immediate sense-perception, and has its knowledge

[1] Berkeley's *Works,* Oxford, 1901, Vol. I, p. 86.

[2] *Ibid.,* I, 22.

only at second-hand, it is in fact not so well fitted as sensation to judge and discriminate. Pure intellect, Berkeley continues, has no jurisdiction in mathematics, for it is concerned only with the operations of the mind, and "lines and triangles are not operations of the mind."[3]

Now this view of the nature of geometry is the direct consequence of Berkeley's early metaphysical doctrine, but it is interesting to note that it also connected itself in his mind with the method of indivisibles maintained by the Italian mathematician Cavalieri. "All might be demonstrated," he says, "by a new method of indivisibles, easier perhaps and juster than that of Cavalerius."[4] What precisely Cavalieri meant by his conception of indivisibles is open to doubt, but it is certain that Berkeley's sympathy would be elicited by his demonstration that quantities are composed of indivisible units, a line being made up of points, a surface of lines, and a volume of surfaces. It is possible, though he is very obscure, that he regarded areas as composed of exceedingly small indivisible atoms of area. Berkeley's conception is very similar to this; but whereas Cavalieri maintained that the number of points in a line is infinite, Berkeley was convinced that no line or surface can contain more than a finite number of points, points for him being *minima sensibilia*. This, then, is Berkeley's "new method of indivisibles."

It will follow that geometry must be conceived to be an applied science. The only pure science will be algebra, for it alone deals with signs in abstraction from concrete things. Geometry may be regarded as an application of arithmetic and algebra to points, i. e., the *minima sensibilia* which constitute the whole of concrete reality, Berkeley admits that it is difficult for us "to imagine a minimum,"[5] but the reason is simply that we have not been accustomed to take note of it separately. In reading we

[3] *Ibid.*, I, 22; cf. I, 14. [4] *Ibid.*, I, 87. [5] *Ibid.*, I, 85.

do not usually notice explicitly each particular letter; but the words and pages *can* be analyzed down to these minimal letters. Similarly, though we are not explicitly aware of the *minima sensibilia*, they do exist separately, and may be analyzed as indivisibles in the complex sense-datum presented to us in perception. Geometry, then, is an applied science dealing with finite magnitudes composed of indivisible *minima sensibilia*.

If this conception of geometry be adopted, it immediately follows, as Berkeley very clearly perceived, that most of the traditional Euclidean geometry must be rejected. (1) In the first place, on the new theory, not all lines are capable of bisection.[6] Only those lines which consist of an even number of points can be bisected. If the number of points composing the line be odd, then (supposing bisection to be possible) the line of bisection would have to pass through the central point. But the point is *ex hypothesi* indivisible; hence the line does not admit of bisection. (2) Again, the mathematical doctrine of the incommensurability of the side and diagonal of the square must be rejected.[7] For since both the side and the diagonal of the square consist of a finite number of points, the relation between these lines will always be capable of exact numerical expression. Berkeley even makes the general statement, "I say there are no incommensurables, no surds."[8] (3) It follows that one square can never be double another, for that is possible only on the assumption of incommensurables. And it also follows that the Pythagorean theorem (Euclid, I, 47) is false.[9] (4) Further, it is no longer possible to maintain that a mean proportional may be found between any two given lines. A mean pro-

[6] *Ibid.*, I, 79, 80.
[7] *Ibid.*, I, 60, 78, 79.
[8] *Ibid.*, I, 14.
[9] *Ibid.*, I, 19.

portional will be possible, on Berkeley's theory, only in the special case where the numbers of the points contained in the two lines will, if multiplied together, produce a square number.[10] (5) Finally, the important work that had recently been done on the problem of squaring the circle is, in Berkeley's view, quite useless. Any visible or tangible circle, i. e., any actually constructed circle, may be squared approximately; and it is therefore time thrown away to invent general methods for the quadrature of all circles.

That his new doctrine necessitated such a clean sweep of important mathematical propositions, most of which had been accepted for hundreds of years, might well have given pause to an even more confident man than Berkeley; for (to take only one instance) apart from its startling theoretical aspects, serious practical difficulties would arise if some lines should prove incapable of bisection. Berkeley therefore suggests that for practical purposes small errors may be neglected. Though we cannot bisect a line consisting of 5 points, we can divide it into two parts, one containing 3 points, the other 2; and, as the *minimum sensibile* is so minute, it makes no practical difference that the lines are only approximately equal. Berkeley was influenced to make this suggestion by the method of neglecting differences practised in the calculus.[11] If differentials, which are admitted to be something, are overlooked under certain circumstances in the calculus, are we not justified in the new geometry, Berkeley asks, in neglecting everything less than the *minimum sensibile*?[12] The resulting errors will be so slight that the usefulness of geometry,

[10] *Ibid.*, I, 14.

[11] *Ibid.*, I, 85.

[12] It might seem that in our approximate bisection of the line we have neglected a whole *minimum sensibile*. But from the point of view of the error involved in each of the resulting parts we are not guilty of that. Each of the parts ought to contain 2½ points. Now each of the lines obtained by the approximate method differs from this by only ½ a point. Hence the error to be neglected in each case is less than a *minimum sensibile*. And this is the condition laid down by Berkeley.

which it must be remembered is a practical science, will not seriously be impaired.[13]

It is of peculiar interest to notice that Berkeley was influenced to neglect small errors, and to justify his procedure, by the example of the differential calculus. For in the *Analyst,* written nearly thirty years later, he vigorously attacked the method of ignoring small errors in the calculus. What a triumph for his opponents in the *Analyst* controversy if they could have seen the *Commonplace Book*!

But though Berkeley made use of the illegitimate method suggested by the calculus, his attitude to the calculus itself was from the first exceedingly critical. And his motive for criticism is not far to seek. If the calculus were sound, then his conception of geometry could not be maintained. For the calculus, whether in the form of Newton's theory of fluxions or Leibniz's method of differentials, rested, Berkeley believed, on the assumption of the existence of infinitely small quantities. Now if these infinitesimals were admitted to exist the significance of his *minima sensibilia* would disappear, and indeed the foundations of his philosophy as a whole would be seriously shaken. For if quantities could be proved to exist which were neither sensible nor imaginable he would need to revise his theory of knowledge altogether. He therefore had every reason to look with critical eyes on the conception of infinitely small quantities.

In the *Commonplace Book* he says nothing of importance with regard to the use to which infinitesimals are put in the calculus. Yet even then he was certainly acquainted with a good deal of the work that had been done on fluxions and differentials. His notes contain references, on matters connected with infinitesimals, not only to Newton and Leibniz but also to Barrow, in whose *Lectiones*

[13] *Ibid.*, I, 78.

opticae et geometricae (1669) was given the chief impulse to Newton's theory of fluxions; to Wallis (1616-1703), whose *Arithmetica infinitorum* (1656) paved the way for the invention of the calculus; to Keill (1671-1721), who, in addition to his *Introductio ad veram physicam* (1702), had written of fluxions in the *Philosophical Transactions* of the Royal Society, and took a prominent part in the famous "Priority controversy" in which he accused Leibniz of having derived the fundamental ideas of the calculus from Newton; to Halley (1656-1742) who in addition to his works on astronomy and magnetism wrote on fluxions in the *Philosophical Transactions*; to Cheyne (1671-1743), whose *Fluxionum methodus inversa* (1703) and *Philosophical Principles of Natural Religion* (1705) gained him admission to the Royal Society; to Joseph Raphson, whose *De Spatio reali seu ente infinito* (1697) contained a definition of the infinitely small, and who was afterwards to write a *History of Fluxions*; and also to two more elementary writers, Hayes (1678-1760) who published in 1704 his *Treatise of Fluxions,* and John Harris whose *New Short Treatise of Algebra....Together with a Specimen of the Nature and Algorithm of Fluxions* (1702) was the first elementary book on fluxions to be published in England. And that he had not confined his reading to English works is proved by his reference to the *Analyse des Infiniments Petits,* and to the controversy between Leibniz and Bernhard Nieuwentijt, a Dutch physician and physicist, which took place in 1694-5 in the pages of the Leipsic *Acta Eruditorum.*[14]

It is clear, then, that even when the *Commonplace Book* was written Berkeley was acquainted with much of the work that had been done in the calculus. But at that time he was not in possession of the arguments which he

[14] The last-mentioned references occur, not in the *Commonplace Book,* but in the essay "Of Infinites" (*Works,* III, 411).

afterwards advanced against it in the *Analyst.*[15] In the *Commonplace Book* he does not venture any criticism in detail of the use of infinitesimals *in the calculus.*[16] What he is concerned to do there is to prove that infinitesimals have no real existence at all.

His line of argument is indicated twice over, and is based on his own metaphysic. For the purpose of his proof he posits two axioms: (I) "No word to be used without an idea," and (II) "No reasoning about things whereof we have no idea." Now we have no idea, Berkeley says, of an infinitesimal. By this he means, if his terminology be translated, that infinitesimals cannot be either objects of sense-perception or objects of representation in imagination. Hence, as we have no idea of an infinitesimal, it is simply a word. Further, according to axiom I, it is a word which means nothing; and, according to axiom II, we have no right to use it in our calculations.

We have now considered in outline Berkeley's attitude, as revealed in the *Commonplace Book,* to contemporary mathematical problems. His willingness to throw overboard the solid achievements of the established geometry simply because they did not accord with an *aperçu* of his own does not encourage us to rate his mathematical ability very highly. Or perhaps it would be truer to say that when he wrote the *Commonplace Book* he had not had time to steady his outlook upon science and the world; and allowance may fairly be made for his youthful dreams of a new idea which was destined to revolutionize the sciences, when we remember that it was only about seventy-three years since Galileo expounded the Copernican theory and thus changed entirely the orientation of astronomy and indeed of science as a whole. Another Copernican change,

[15] Some of his remarks show that he was at that time, far from understanding its principles and methods (Cf. *Commonplace Book,* I, 84f).

[16] But there is some criticism of the calculus itself in the essay "Of Infinites" (*Works,* III, 411). And cf. *Commonplace Book,* I, 83-86.

he believed, was not impossible; and in any case he was inclined to think that the wonderful mathematical renaissance of the previous few decades had, among all its triumphs, grown not a few excrescences which it would do no harm but much good to pare off. What he really wished to do was to examine the logical basis of mathematics. He did not advance very far in the *Commonplace Book,* but it was part of what he attempted, and with greater success, in the *Analyst.* To the argument of the *Analyst* we now turn.

The *Analyst* was published in 1734. It is a curious work, and though its purpose is ultimately theological rather than mathematical, it gave rise to a mathematical controversy which lasted for several years and produced more than thirty controversial pamphlets and articles. We have no concern with the theological argument of the *Analyst,* but before passing to consider its mathematical importance, it may be well to mention that the essay is primarily intended as a defense of Christianity, and that Berkeley, acting on the principle that the best defense is in attack, criticizes the foundations of mathematics on the same lines as those on which Christianity had been opposed by "mathematical infidels." In reply to the criticism that the dogmas of Christianity are mysterious and incomprehensible, Berkeley maintains that mathematics, universally admitted to be the most demonstrable department of human knowledge, is, in that regard, in precisely the same position as Christianity. For it also makes use of mysterious and incomprehensible conceptions, e. g., fluxions and infinitesimals. If mathematicians accept mystery and incomprehensibility in mathematics they have no right to object to it in Christianity. This is the kernel of Berkeley's argument.

Berkeley is often regarded, but quite unjustly, as an enemy of the infinitesimal calculus. In reality he had no objection in the world to the calculus as such. What he

did was to submit its logical basis to a searching examination. He criticized the conception of infinitely small quantities, which were at that time vaguely conceived as neither zero nor finite, but somehow in an intermediate state. They were said to be "nascent" and "evanescent" quantities, not quite nothing and not quite anything. It was against this "vague, mysterious and incomprehensible notion" that all Berkeley's attacks were directed; and as soon as it was clearly pointed out by one of the parties to the controversy, Benjamin Robins,[17] that the calculus did not necessarily involve this conception of infinitely small quantities, but might be demonstrated by the methods of limits, the controversy was abandoned by Berkeley. He had replied to his other critics, such as Jurin of Cambridge ("Philalethes Cantabrigiensis") and Walton of Dublin, because these mathematicians persisted in trying to defend the conception of infinitely small quantities. But as soon as it became clear, and Robins was the first to make it so, that that conception was not essential to the calculus, the controversy lost interest for Berkeley. For the method of limits, as he seems to have realized, is not incomprehensible; and therefore an attack on it would not have enabled him to use his *tu quoque* argument, and would thus no longer serve his purpose, which, it must be remembered, was primarily theological.[18]

[17] Robins's contributions to the controversy were contained in his *Discourse concerning the Nature and Certainty of Sir Isaac Newton's Methods of Fluxions, and of Prime and Ultimate Ratios* (1735), and in a series of articles in the *Republic of Letters* in 1736 and in the *Works of the Learned* in 1737.

[18] The course of the *Analyst* controversy, so far as Berkeley was concerned, was as follows. In 1734 the *Analyst* appeared. It was almost immediately attacked by Jurin in an anonymous tract entitled *Geometry no Friend to Infidelity; or a Defence of Sir Isaac Newton and the British Mathematicians.* To this Berkeley replied in *A Defence of Free-Thinking in Mathematics,* published in March, 1735. To this reply Jurin wrote a rejoinder which was published in July of the same year. Berkeley took no notice of it.

Berkeley had another critic. This was Walton of Dublin, who produced in 1735 a *Vindication of Sir Isaac Newton's Fluxions.* It was replied to in an appendix to the second edition of *A Defence of Free-Thinking in Mathematics.* Walton replied, and Berkeley then published his *Reasons for not replying to*

But though his motive in writing the *Analyst* was theological, the chief importance of the book, as we must now try to show, is mathematical. It is, indeed, an able treatise on the logic of mathematics. Berkeley saw that the brilliance of the rapidly accumulating results attained by means of the calculus had tended to put into the background the question of its logical basis and the validity of the methods employed by it. And he did good service to mathematics by the publication of the *Analyst,* for he forced upon mathematicians the investigation of the logical basis of the new mathematics. "I have no controversy," he says, "about your conclusions, but only about your logic and method. . . . I beg leave to repeat and insist that I consider the geometrical analyst as a logician, i. e., so far forth as he reasons and argues; and his mathematical conclusions, not in themselves, but in their premises, not as true or false, useful or insignificant, but as derived from such principles and by such inferences."[19] As a direct result of this investigation originated by Berkeley two highly important principles were firmly established, (1) that the calculus must be grounded on the method of limits, and (2) that the then current conception of infinitely small quantities must be abandoned.

These points will become clear if we examine Berkeley's criticism of Newton's theory of fluxions. In our investigation there are three main questions which we must ask. (1) Is Berkeley's criticism of Newton valid? (2) Is his criticism of current conceptions of infinitesimals

Mr. Walton's Full Answer. All this took place in 1735. Walton issued a rejoinder, but Berkeley took no further part in the controversy.

It is noticeable that Berkeley participated in the controversy vigorously until Robins's book appeared. After that he said not a word. The reason is, as we have suggested, that Robins showed that infinitesimals are not essential to the calculus. Berkeley must have been convinced by his arguments, and therefore realized that it was no longer possible, from his point of view, to take part in the controversy.

[19] *The Analyst,* § 20.

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sound? (3) Did he really expose any fallacies in the calculus?

1. First, then, we must consider whether Berkeley is successful in his criticism of Newton. There is one special point in Newton's theory which must be examined with some care, for upon it depends the applicability of Berkeley's criticism. The question is this. Did Newton really use the conception of infinitely small quantity (in which case he would be exposed to the full force of Berkeley's arguments), or was his method really that of limiting ratios (in which case Berkeley's criticisms would be directed, so far as Newton is concerned, against a man of straw)?

It is often held that Newton never used the conception of infinitely small quantity, but it was conclusively established by De Morgan that this conception does appear in some of his works. De Morgan maintains that until the year 1704 when his *Opticks* was published Newton did believe in infinitely small quantities. "In Newton's earliest papers," writes De Morgan, "the velocities are only differential coefficients: when A changes from x to $x + o$, B changes from y to $y + oq/p$, the velocities being p and q. Those terms in which o remains are 'infinitely less' than those in which it is not, and are therefore 'blotted out.' And 'those terms also vanish in which o still remains, because they are infinitely little.' "[20] Again, in the first edition of the *Principia,* published in 1687, fluxions are founded on infinitesimals, moments being regarded as infinitely small quantities. De Morgan confirms this by relevant quotations from Newton's *Method of Fluxions,* written in the period 1671-1676) and his *Quadratura Curvarum,* which was originally written about the same time, though

[20] "On the Early History of Infinitesimals in England," *Philosophical Magazine,* 1852, IV, 322-3.

it was not published till later. So far, Newton certainly made use of the conception of infinitely small quantity.

But in 1704 the *Quadratura Curvarum* was issued in an appendix to the *Opticks*. It contained a new preface with some important statements regarding infinitesimals. "I here consider mathematical quantities," Newton says, "not as consisting of minimal parts, but as described by continuous motion."[21] "I was anxious to show that in the method of fluxions there is no need to introduce into geometry figures infinitely small."[22] Now Berkeley was well aware that the conception of infinitesimals had been thus disclaimed by Newton. In the early essay "Of Infinites" he says, "Sir Isaac Newton, in a late treatise,[23] informs us his method of fluxions can be made out *a priori* without the supposition of quantities infinitely small."[24]

But in 1713, when the second edition of the *Principia* was published, Newton again admitted, though very obscurely, infinitely small quantities.[25] From all this we may conclude that while Newton did not give exclusive adhesion to the method of infinitesimals, yet the conception of infinitely small quantity does occur in his writings prior to 1704, and though it was renounced in that year it reappears in the second edition of the *Principia* in 1713. It therefore follows that Berkeley's criticism is pertinent. Newton, we have decided, did maintain the existence of

[21] "Quantitates mathematicas non ut ex partibus quam minimis constantes, sed ut motu continuo descriptas hic considero."

[22] "Volui astendere quod in methodo fluxionum non opus sit figuras infinite parvas in geometriam introducere."

[23] This refers to the *Quadratura Curvarum*. Berkeley's "Of Infinites" was written about 1706-7.

[24] Berkeley's *Works*, III, 412.

[25] This point has been regarded as open to doubt. It depends on Newton's definition of moment. The definition is stated somewhat differently, but very obscurely in both cases, in the first and second editions of the *Principia*, in Book II, lemma II. But Edleston cites a letter from Newton to Keill written in May, 1714, in which he says explicitly, "Moments are infinitely little parts" (J. Edleston, *Correspondence of Sir Isaac Newton and Professor Cotes*, p. 176). This seems to be conclusive evidence that Newton still clung to infinitesimals.

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infinitely small quantities, and it is against these that Berkeley argues.

Berkeley points out a serious inconsistency in Newton's conception. He shows that at one time Newton admits that infinitely small moments may under certain circumstances be altogether omitted in calculation. Against this admission he arrays Newton's declaration that in mathematical operations even the smallest errors must not be overlooked. Now the former of these statements is made by Newton in the *Principia* and the latter in the *Quadratura Curvarum*. The two are obviously inconsistent. Berkeley's critics in the controversy tried to defend Newton in various ways, but neither of them dared to admit, even if they perceived it, that the inconsistency was due to a change in Newton's system. In the *Principia*, holding a conception of infinitesimals, he is forced, precisely as the continental exponents of the differential calculus were forced, to admit that infinitely small quantities are negligible in calculation in comparison with those of finite magnitude. On the other hand, in the *Quadratura Curvarum*, having renounced infinitesimals, he is free to assert that even the smallest errors cannot be permitted. Benjamin Robins was the first of Newton's defenders to see clearly that the systems are different, and that if Newton's position is to be seriously defended it will be necessary to admit frankly the change of system, and to maintain that for Newton the really fundamental method is the method of limits.[26]

[26] Berkeley has been accused of bad faith in advancing this criticism. He must have seen, it is argued, that the Newton of the *Principia* was in a different position from the Newton of the *Quadratura Curvarum*, and therefore he was not justified in arraying the statements of these two periods against one another as evidence of present inconsistency (cf. A. De Morgan, *op. cit.*, p. 329). But such an argument overlooks two or three very material facts. The first is that Newton himself nowhere explicitly admits a change of system; in fact he seems anxious to conceal that such a change had taken place. Further, with the exception of Robins, Newton's followers were far from clear whether or not a change had taken place; and, in any case, as we have seen, Newton seems to have returned to the conception of infinitely small

This is what Robins did, and it has come to be realized that the conception of limits forms the true logical basis of the calculus. Berkeley's general criticism of Newton is perfectly valid, and it was largely owing to his objections that the difference between the two methods came to be fully appreciated, and that eventually a method of limits akin to that of Newton was established as the foundation of the calculus.

But in two respects Berkeley was unfair to Newton.

a. He never lets his reader know that Newton used the method of limits at all, and always speaks as if Newton had always held that the method of infinitesimals was essential to his theory of the calculus. Now the truth is, as Robins pointed out, that everything of fundamental importance in Newton's work is perfectly consistent with the method of limits.

b. He gives Newton no credit for his doctrine of continuity. Newton's infinitesimals are, after all, never so self-contradictory as those of Leibniz or even of his own followers. His infinitely small quantities are not, like Leibniz's differentials, discrete particulars. The Leibnizians hold that the "difference" of a line is an infinitely little line, the "difference" of a plane an infinitely little plane, and so on. And Newton's own followers used the conception of infinity in an equally rash way. Thus De Moivre regards the fluxion of an area as an infinitely small rectangle; and Halley, to whom Berkeley refers in the *Commonplace Book,* speaks of infinitely small *ratiunculae* and *differentiolae* in much the same way as the Leibnizians. Hayes, again, another follower of Newton to whom Ber-

quantity in 1713. Now, the *Analyst* was not published till 1734, and at that distance of time Berkeley may quite well have regarded Newton's renunciation of infinitesimals in 1704 as a temporary aberration. In that case he would be perfectly justified in his criticism.

[27] For an appreciation of Benjamin Robins, see Prof. G. A. Gibson's review of Cantor's *Geschichte der Mathematik* in *Proc. Edin. Math. Soc.,* 1899, pp. 20ff.

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keley also refers, maintains the conception of infinitely small quantity with much frankness. "Magnitude," he says, "is divisible *in infinitum.* Now those infinitely little parts, being extended, are again infinitely divisible; and those infinitely little parts of an infinitely little part of a given quantity are by geometers called *infinitesimae infinitesimarum* or *fluxions of fluxions.*"[28] But Newton himself does not speak in that way. He never forgets that his whole system is based on the continuity of motion. Lines are generated by the motion of points, planes by the motion of lines, and solids by the motion of planes. Fluxions are strictly the velocities of the generating motions. And the continuity of motion, generating lines, surfaces, and the like with varying velocities involves the conception of prime and ultimate ratios. To this aspect of Newton's theory Berkeley seems to be blind.

2. Having considered the respects in which Berkeley's criticism of Newton is valid, we may now proceed to ask whether his criticism of infinitesimals in general will bear examination. The general criticism of infinitesimals consists of two arguments, only one of which seems to be sound.

a. Berkeley argues—to take first the contention that seems unsound—that infinitesimals are impossible because imperceptible. An infinitely small quantity cannot be the object either of sense-perception or of imagination, and in accordance with the formula *esse est percipi* it can therefore have no existence. "As our sense is strained and puzzled with the perception of objects extremely minute, even so the Imagination, which faculty derives from Sense, is very much strained and puzzled to frame clear ideas of the least particles of time, or the least increments generated therein; and much more so to comprehend the

[28] *A Treatise of Fluxions,* 1704. Quoted by A. De Morgan in *Essays on the Life and Work of Newton,* edited by P. E. B. Jourdain, Chicago and London, 1914, p. 91.

moments, or those increments of the flowing quantities *in statu nascenti,* in their very first origin or beginning to exist, before they become finite particles.[29]

Now, this argument is simply at the level of picture-thinking. It does not follow that what we are unable to perceive in sense-perception or to represent in imagination is non-existent. At one time Berkeley's new principle would have necessitated this argument, but when the *Analyst* was written he had outgrown the cruder form of his early theory, and in his doctrine of notions he admitted that we can have knowledge which comes neither through sense nor imagination. He was thus prepared to allow that we might have real knowledge not sensuous in its origin. His retention of the argument here is a sign that he was not completely emancipated from his early sensationalism.

b. Berkeley's second general argument against infinitesimals is perfectly sound. He points out that the conception of the infinitely small, whether in the form in which it appears in Newton and his followers or as maintained by Leibniz, is impossible. It is impossible because it is self-contradictory. Whether we regard infinitesimals with Leibniz as differences, i. e., as infinitely small increments or decrements, or with Newton as fluxions, i. e., velocities of nascent or evanescent increments, they involve in their nature an ultimate contradiction. On the one hand, an infinitesimal seems to be something, for otherwise it would not be used in mathematics; but on the other it seems to be nothing, for mathematicians say it may be neglected in calculations without affecting the accuracy of their results. Sometimes it is called a nascent quantity, i. e., one which has left being nothing, but has not yet quite become anything; at other times it is called evanescent, i. e., a quantity which is still something but is tending to be almost

[29] *The Analyst,* § 4.

(though not quite) nothing. This conception, Berkeley insists, is ultimately incomprehensible and contradictory. His argument here is, of course, perfectly sound. Infinitesimals, conceived in this vague and loose way, have now, very largely owing to the process of criticism initiated by Berkeley, been entirely extruded from the calculus.

3. The last problem which we set before ourselves is this. Did Berkeley, apart from stimulating the investigation of the logical basis of the calculus, expose any real errors in it? From his arguments in the *Analyst* it would seem that two main errors affect the calculus. Berkeley maintains that (*a*) any attempt to demonstrate the value of a fluxion involves the violation of ultimate logical principles, and (*b*) the maxim that infinitely small errors compensate one another is vicious. A word or two must be said on each of these points.

a. In order to prove the illogicality of the methods of determining the value of fluxions, Berkeley examines in some detail the two independent demonstrations given by Newton. In the *Principia* Newton gives a geometrical proof, in the *Quadratura Curvarum* an algebraic one. In each case, Berkeley seeks to show, a closely analogous error is committed.

Take first Newton's geometrical demonstration. We wish to find the fluxion of the rectangle AB generated by the continuous motion of one side upon the other. Let the moments or momentaneous increments of A and B be *a* and *b* respectively.

When the sides of the rectangles are each diminished by half their moments, the rectangle becomes

$$(A - \tfrac{1}{2}a)(B - \tfrac{1}{2}b)$$

$$\text{i. e., } AB - \tfrac{1}{2}aB - \tfrac{1}{2}bA + \tfrac{1}{4}ab.$$

Similarly, when the two sides are increased by half their moments, the rectangle becomes

$$(A + \tfrac{1}{2}a)(B + \tfrac{1}{2}b)$$
$$\text{i.e., } AB + \tfrac{1}{2}aB + \tfrac{1}{2}bA + \tfrac{1}{4}ab.$$

Subtract now the former rectangle from the latter, and the remainder is $aB + bA$. This remainder is the moment of the rectangle generated by the moments a, b of the sides. Such is Newton's proof.

In criticism of it Berkeley maintains that the natural and direct method of obtaining the moment of the rectangle AB, when the moments of its sides are a and b, is to multiply into one another the sides increased respectively by their *whole* moments.[30] The moment of the rectangle is therefore

$$(A + a)(B + b) - AB,$$
$$\text{i. e., } AB + aB + bA + ab - AB,$$
$$\text{i. e., } \qquad aB + bA + ab.$$

This, Berkeley says, is the true moment or increment. It differs from that obtained by Newton's proof by the quantity ab. Now, as it was essential for the method of fluxions to eliminate the term ab, Newton and his followers said that it was so infinitely small that it could simply be neglected. But against this defense Berkeley quotes Newton's own words, "In rebus mathematicis errores quam minimi non sunt contemnendi."[31]

Berkeley shows that Newton's algebraic proof also rests on illegitimate assumptions.[32] In this demonstration we are given the uniformly flowing quantity x, and it is required to find the fluxion of x^n.

Suppose that x, in process of constant flux, becomes $x + o$, then x^n becomes $(x + o)^n$. Expanding this by the method of infinite series we get

$$x^n + nox^{n-1} + \tfrac{1}{2}n(n-1)o^2x^{n-2} + \ldots.$$

(i. e., the increment of x^n is $nox^{n-1} + \tfrac{1}{2}n(n-1)o^2x^{n-2} + \ldots$).

[30] *The Analyst*, §§ 9ff.

[31] *Opticks*, Introduction to *Quadratura Curvarum*.

[32] *The Analyst*, §§ 13ff.

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It follows that the increments of x and x^n are to each other as o to $nox^{n-1}+\frac{1}{2}n(n-1)o^2x^{n-2}+\ldots\ldots$
or, dividing by the common quantity o,

$$\text{as } 1 \text{ to } nx^{n-1}+\tfrac{1}{2}n(n-1)ox^{n-2}+\ldots.$$

Now, "let the increments vanish," and the last or limiting proportion is $1 : nx^{n-1}$. The ratio of the fluxion of x to that of x^n is as 1 is to nx^{n-1}.

Berkeley points out that this reasoning is illogical. If we say, "Let the increments vanish," we imply that the increments are really nothing, seeing that they are negligible. But we are enabled to arrive at the proportion between the fluxions only by assuming that the increments are something. Berkeley accordingly maintains that it is illogical to reject the increments, and still retain an expression, i. e., the proportion of the fluxions, obtained by means of them. If we let the increments vanish, we must also in consistency let everything derived from the supposition of their existence vanish with them.

This criticism Berkeley supports with a lemma, which he states as follows, "If, with a view to demonstrate any proposition, a certain point is supposed, by virtue of which certain other points are attained; and such supposed point be itself afterward destroyed or rejected by a contrary supposition; in that case, all the other points attained threby, and consequent thereupon, must also be destroyed and rejected, so as from thenceforward to be no more supposed or applied in the demonstration."[33]

b. Berkeley goes on to urge that, even though correct results are attained by the application of the method of fluxions, that does not validate the method as method. That the conclusion of a syllogism is true does not necessarily imply that the process of reasoning is correct. The conclusion may be true, and yet logical errors may have been

[33] *The Analyst*, § 12.

committed in the process of proof. It is possible to reach a true conclusion from false premises by erroneous reasoning. One error compensates the other, and thus, though the conclusion is true, the logic is faulty. Precisely similar is the case of the calculus. True conclusions may be attained by it, and results of great practical value may be achieved, but its method is unsound because it is based upon the illogical principle of the compensation of errors.

These are the main arguments advanced by Berkeley in the *Analyst*. In the controversy which ensued all the points that he raised were traversed and re-traversed, with the result that (1) the vague notion of infinitely small quantity was abandoned, (2) the method of limiting ratios was firmly established, and (3) the principle of the compensation of errors was seen to be inconsistent with the logical foundation of the calculus.

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INFINITY AS METHOD.

AFTER the mathematical theory of assemblages had been developed, the logic of infinity entered upon a new stage. If we cannot as yet determine the final form of the doctrine, we can, at least, see at the present time in what direction it is tending. After the discovery of the calculus of infinite aggregates and after the establishment of different exactly distinguished kinds of infinity, the perpetual problem as to the potential or actual character of mathematical infinity seems to incline toward a solution in terms of actuality.[1] But this actuality seems to belong rather to the methodological than to the quantitative character of infinite aggregates; *it is a property of methods, not of quantities,* and expreses merely a peculiar system of laws and principles logically working in and upon finite magnitudes. From this point of view infinity cannot be regarded as a kind of quantity, but rather as the *type of structure* of certain quantities;—it does not pass the "limits of our possible experience" as if it were an expression for something "we never could find on sea or land." It is one of the constructive laws of our normal

[1] G. Cantor, "Mitteilungen zur Lehre vom Transfiniten" (*Zeitschrift für Philos. und philosoph. Kritik,* Vol. 91, pp. 81 ff. Comp. Vol. 88, pp. 224 f). Couturat, *De l'infinie mathématique,* pp. x, 213-256; 488-563. Royce, *The World and the Individual,* Vol. II, pp. 554 ff. Spaulding, *Defense of Analysis* (New Realism). H. Lanz, "The Problems of Immortality" (in Russian language, *Logos,* 1913). Gavronsky, *Das Urtheil der Realität* (Dissert., Marburg). Cohen, *Logik der reinen Erkenntniss,* pp. 102 ff. The actual character of infinitely small elements has been mathematically established in Veronese's *Grundlagen der Geometrie von mehreren Dimensionen* (transl. from the Italian). Methodological character of infinity without acceptance of its actuality is emphasized by Brown, *Intelligence and Mathematics* (Creative Intelligence).

experience, perfectly incorporated within every finite object. If two parallel lines do not cross each other in some infinitely distant point, the geometrical structure of our triangles, circles and the like will be different. This mysterious "point" has an *influence* upon finite geometrical forms which is somehow manifested in their inner relations. But is it the "point" that makes the relations what they are? Why not invert our presumption? Why not suppose the structure of finite relations to be prior, infinity being a result of them?

If we can discover all the methodological principles that have to do with infinity, and investigate the reflection of "endlessness" into the world of finite magnitudes, we shall be led to the conclusion that infinity is only an aspect of our normal experience, a property of finite things; that the infiniteness of "space" is nothing but a character of "single spaces," the infinity of series only a property of certain magnitudes. Mathematically however every "property" is an expression of some constructive method. Accordingly the old question as to whether infinity is a category of the qualitative or the quantitative sort, appears unexpectedly in a new light: it seems to be prior to both sorts, being a complex of methods rather than a *quale* or even *quantum*. Those peculiar "qualities" which belong as much to the spatial point as to the instant of time, as much to the sum total of algebraic numbers as to every general concept in its infinite integrating function, lose thus their metaphysical mystery. A point without determined "position" has no peculiar "quale"; taken abstractly it is pure nonsense. This mystical "quale" properly belongs to a point only in so far as it is a *point of a curve. It is a moment in the continuous change of direction,* expressed in the methodical exactitude of a derivative function, not a mysterious "part" of space without extension. For mathematically infinity is always connected with exactly deter-

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mined methods of operation, in principle different from addition or division, even in their indefiniteness. This difference in the methods of operation makes every instance of mathematical infinity actual in spite of the impossibility of its production by any arbitrary long process of addition or division. The method of integrals, for instance, perfectly achieves a task which the ordinary methods of addition and division can never fulfil: it leads us to apparent "qualitative" new results,—sometimes to new kinds and unexpected generalizations of number.

The matter in question has often been discussed in mathematical and philosophical literature; but it may be of interest to follow the development of the methodical meaning of infinity as well in the deepest metaphysical speculations of antiquity as in the exactest mathematical achievements of our own epoch. We shall find, then, that this aspect, consciously or unconsciously, is predominant in every "case" and every "kind" of infinity. Thus will become clearer and clearer that positive moment, implied in the earliest unmethodical negations of finiteness, and the "dark" quality as well as the unintelligible "endlessness" of the infinite will be seen to be but concentrated expressions for certain methods of operation involved. This point of view may, possibly, remove the usual distrust and disgust of mathematicians for the "actual infinite."

Qualitative infinity. The infinite is not "not-finite." From the first historical use of the term by Anaximander, the ἄπειρον purports to be a *positive* principle for the explanation of all single and separate "quales," being far from an equivalent for pure nothingness or endlessness. As an ἀρχή or first principle of being, τὸ ἄπειρον is "infinite in a positive sense because it expresses a belief in and a demand for a "different nature" (ἑτέραν τινὰ φύσιν) required for the genesis and derivation of "beings" (ὄντα) —a kind of primitive generating ἕτερον (ἕτερον τι τῶν

στοιχείων). To be principle is essential for the ἄπειρον; it is nothing but principle or basis (ὑποκείμενον) and has no meaning apart from its "consequences," innate or implicit in it.[2] The *infinitum* is possible and intelligible only in its relation to finite objects, in its logical activity as constructive and productive principle of finite results. It is prior to everything, because τὸ ἀόριστον πρὶν ὁρισθῆναι,[3] and it cannot be constructed by the successive addition or condensation of all things (μεταβολὴ τῆς ὕλης) because its nature is positive, simple and indivisible.[4]

These vague speculations of the earliest Greek philosophy do little more than mark out the field for later analysis. Nevertheless they clearly indicate the primary phase of the problem, in which the finite being begins to look for its origin beyond itself—*in infinitum*. The non-finite, that which is to explain the finites, defines itself as the *problem* of "in-finite." The dim historical previsions, concealed in this definition, soon reveal their positive purport. Aristotle—that scholastic lover of subtle and sterile distinctions—in our present problem brought out a discrimination of great importance. He set up two different concepts of infinity which might have been of a great historical influence and systematic fruitfulness. In his Metaphysics we meet with the clear distinction between the potential (τὸ δυνάμει ἄπειρον) and the actual infinity (ἐνεργείᾳ ἄπειρον). The potential infinite persistently remains within the confines of finite processes and means nothing but the possibility of continuing these processes indefinitely. It remains in the methodical power of the measure, in the

[2] Ἁμαρτάνει οὖν, τὴν μὲν ὕλην ἀποφαινόμενος, τὸ δὲ ποιοῦν αἴτιον ἀναιρῶν· τὸ γὰρ ἄπειρον οὐδὲν ἄλλο....ἔστιν. *Anaximander Milesius* (Neuhaeuser, p. 6). Comp. Simplicius, *Phys.*, p. 32, β: ἐνούσας γὰρ τὰς ἐναντιότητας ἐν τῷ ὑποκειμένῳ ἀπείρῳ. Therefor the ἄπειρον being a different nature, it is not apart from reality. These sentences of Anaximander may be the first indication of the latter metaphysical speculations of Fichte and Schelling, according to which doctrine the "principle" without appearance is nothing; it appears necessarily and exists only in its manifestation in the world of finites; comp. the author's "Fichte and his Doctrine of the Absolute" in Russian).

[3] Aristotle, Met. I, 8, p. 989. [4] Neuhaeuser, *Anaximander Milesius*, p. 44.

field of its logical activity; it is adjusted for measurement and adapted to its "being gone through," but cannot be "gone through" because of the absence of the end.[5] This kind of infinity is only possible as a finite *variable* quantity in process of augmentation or diminution, something remaining always "beyond."[6] This purely negative concept of endlessness, unintelligible and contradictory as it is, has been generally acknowledged by mathematicians and is still current in that branch of science. The other logical type or variant of infinity given by Aristotle is without mathematical import or value and was meant to satisfy the purely logical interest in the notion of οὐσία. This ἐνεργείᾳ ἄπειρον (*infinitum actu*) is apparently the historical source of the "qualitative infinite"—an infinite transcending the problem, the function and the methods of measurement, and as remote from any implication of process as melody is inaccessible for sight. This kind of infinity *being beyond the concept of measure* is thus without "extension"; it has no "middle," no "above," no "below";[7] it does not consist of "parts" and is in the strict sense indivisible.[8] We must give over enumeration if we want to grasp infinity *in actu*; it is impossible to understand or to construct it in terms of continued recurrence of finite elements; in a word actuality marks in this primitive stage the creation of a new quality, the elevation of the mind to an entirely different level expressible only in terms of "ideality." Historically the meaning of ideality is connected inseparably with infinity[9] because to consider any fragment of reality under the aspect of ideality means to consider it as an instance of universal conformity to law. "Ideality" is the explanation of infinity *in actu,* or the resulting "quale" of infinity.[10]

This new qualification has been for many centuries the

[5] Aristotle, Met., κ, 10. [6] Aristotle, Phys., γ, 6. [7] Aristotle, Phys., γ, 5.
[8] Aristotle, Met., κ, 10. [9] Comp. Schelling, *Bruno, S. W.*, I, 4, pp. 342 ff.
[10] Hegel, *Logik, S. W.*, III, pp. 165, 171 ff.

main subject of philosophical reflection. In terms of infinity Plotinus defines his overtemporal realm of creative intelligence, where every "part" possesses the same "power" as the whole.[11] In the same terms Spinoza constructs his concept of substance, by which the must have meant to express neither more nor less than the logical inevitability of all the laws of nature.[12] In medieval literature also we meet with a very instructive instance of the qualitative infinity. I mean the doctrine of eternity. Thus Anselm constructs his concept of God in terms of "eternal truth" by the method of time-negation;[13] and this eternity is no potential or repetitive infinity of time. It transcends all lapse of time; it is an "indivisible unity" beyond time,[14]—*tota sibi praesens*.[15] This is the original source from which the modern concept of *Geltung* or Ideality has been derived. And what is of more importance, Anselm in his explanation of "over-temporality" goes further perhaps than Bolzano, Husserl, or even Bradley. According to his doctrine, the irrelevance to time limits not only produces a peculiar quality but is caused by negation and suspension of all the methodical means used for the explanation of temporal reality. *The positive ground for this new quality is discovered in the conformity to a new law.*[16] which has found its positive expression and justification in the laws of Logic.

Thus the definition of infinity as "quale" reveals its positive value when applied to the problems of pure logic.

[11] Plotinus, Enneades, III, 8, 8; VI, 9, 6. Comp. Henry Lanz, "Speculative Transcendentalism in Plotinus" (printed in Russian in the Journal of Ministry of National Education, 1914, I, 2.

[12] Comp. Wenzel, *Die Weltanschauung Spinozas.*

[13] Sancti Anselmi opera omnia (*Patrologiae cursus completus* T., 159), Monologium, pp. 160, 198 ff; Proslogion, pp. 235, 237, 239; Dialogus de veritate, p. 479.

[14] Anselm, Proslogion, p. 237. [15] *Ibid.*, p. 238.

[16] Anselm, *Monologium*, p. 175: "Procul dubio summa substantia, quae nulla loci vel temporis continentia cingitur, *nulla eorum lege constringitur.*" Comp. *ibid.*, p. 166: "Ita uno modo, *una considerationе* est, quicquid est essentialiter." Comp. *ibid.*, 184-185. Thus according to Anselm's conception the essentia or idea is not apart from reality but a certain "consideration" of it, the result of the methodical application of certain laws.

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Every logical content, every proposition in its value and relative truth can be regarded as an *instance of infinity,* because it marks the abandonment of the primitive attitude of enumerating the single cases—a turning from the sheer pluralism of sense-perception to the universality of method established first in Plato's "idea." The result was a different kind of logical operation, absorbing in the *process of deduction* all the possible cases *in infinitum,* instead of their enumeration *in indefinitum. The infinity implied in every logical concept is "actual" not because all its single cases have been enumerated, but because enumeration is no longer significant or serviceable.* In this sense "actual" infinity is simply the expression of a generality implied and employed in all use of "general concepts." It is no inherent or peculiar quality of "ideas" enabling us to apply to them our deductive, dialectical or transcendental methods; on the contrary by our methods we fashion our "ideas" into a logical form and adaptability in which they have for us the semblance of "transcendent entities."

Our modern logic is a positive system of methods, laws and categories which has grown out of these metaphysical speculations concerning eternal ideas, substances, God, *absoluta* and the like. It is an historical outcome of the simple resolve to consider the separate cases not in their plurality but in their systematic totality. So considered, they inevitably stand revealed as infinite logical complexes. Their being instances of a qualitative infinity is nothing but the expression of what they are as instances of a methodical (in a large sense deductive) thinking, and the peculiar quality of logical concepts, expressed in their eternality, overtemporality and so on, is nothing but a peculiar kind of operation with such complexes which justifies them and gives them a definite meaning. They have no existence for us until our methods of dealing with our world have made them seem to exist. Whereupon we say metaphys-

ically, "they exist in no space or time," "they have but an intentional being." The peculiar property or power of logical complexes to embrace an infinity of single cases compels no recognition of a mysterious realm of truths or ideas, beyond reality. It is a natural consequence of our methodological emphasis upon the significant proportions and relations of the finite cases ("The essence is immanent to the appearance,"[17] "objectivity is created by consciousness," "the mind prescribes its law to nature").

Critics of the potential infinity. The usual explanation of infinity in mathematics consists in the assertion that the true infinite is nothing but a symbol expressing the possibility of continued counting or measurement—in a word "potential" infinity.

We may urge against this "subjective" principle of explanation the general objection, that our process of counting does not belong to the objective value or logical purport of any mathematical relations,—it can explain nothing, it can prove nothing with regard to them: it is absolutely irrelevant to their logical constitution or value, and can play no part in their mathematical establishment. The relations which govern in this operation, as executed by our mind, are psychological or epistemological relations which can have nothing to do with arithmetic or with the theory of aggregates. The constructive principle or method of arithmetic excludes by its essential purport all influence of consciousness so that what is impossible for consciousness may be quite possible in principle. This *elimination*

[17] The "essence" by Spinoza—perhaps the most important instance of the qualitative infinity—is based on the same methodological ground; to regard anything in the essence, as a modus of substance, means to regard it *sub specie aeternitatis, as a moment in the deductive development of the system;* everything *is* "substance" in so far as it is an instance of "method," i. e., in so far as it has truth (*sub specie veritatis*) and can be proved. The "essence" does not point out a different thing provided with special qualities, side by side with its real appearance, but the essential relation *within* the appearance itself, its conformity to the immanent law, *its ability of being proved, its position in the system of deduction.*

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of consciousness belongs in fact to the logical intention of all theory as such and is of the essence of all reasoning.[18]

This general proposition has an application to our present discussion. To base the concept of infinity on the ground of mental possibilities or processes means the same as to construct it without any ground, because the recurrence to the process as such has no logical value; the whole reasoning represents a very simple example of *quaternio terminorum.* What properly plays a part in the constructive definition of infinity is not the process as such (as executed in a finite time), but the inner methodical principle of it;[19] the process itself is going on according to this principle, changing no element or item of its logical content or axiomatical formulation. It would be absolutely meaningless to say: the principle does not define the totality of a certain series, because we are unable to stop in our process (*quaternio terminorum*). On the contrary: we cannot rest in our process because the principle does not permit us to rest, because it gives no guidance or indication as to a particular point of absolute rest. The relation must be reversed: it is the logical nature of infiniteness which gives our processes of counting or measuring indefinitely large or small, not contrariwise. Thus the *infinitum potentiale* may be called groundless infinity; the process of its construction is based on a well-known logical fallacy.

[18] The general position here indicated has been elsewhere worked out by the author. Comp. "Das Problem der Gegenständlichkeit in der modernen Logik" (Ergänzungsheft d. Kantstudien, No. 26, 1912); "Fichte und der transcendentale Wahrheitsbegriff." It has also been developed in certain writings of the author in the Russian language (in "Logos" and "Problems of Philosophy and Psychology").

[19] From this point of view may be profitably discussed a very old doubt of the sceptics, that for the purpose to know the infinite "we" must have an infinite capacity, and since we do not have any, we are unable to know anything about infinity. Pascal says, for example: "....il ne faut pas moins de capacité pour aller jusqu'au néant que jusqu'au tout; il la faut infinie pour l'un et l'autre." (*Pensées*, I, p. 82. Nouvelle éd. par Brunschvicg). But as we have seen, any of our "capacity" does not belong and does not have any logical influence upon the content known; therefore we don't need to have an infinite

On the other hand, the mathematical possibility of setting up a limitation in counting ("to construct the concept of number") is dependent upon a long series of special, and only for that purpose inevitable, pre-suppositions, which shall axiomatically define the meaning of the end. After the infinite aggregates had been mathematically defined by means of equivalence between part and whole, a long series of presuppositions was required to construct the finite numbers. The principles of enumeration, established in Dedekind's system of arithmetical axioms by means of "similar representation" and the concept of "chain," are similarly limitations added to Cantor's "axiomatic." Every aggregate of elements may be a "number" not by itself, but only in so far as a certain principle of representation is used; the same aggregate might be ordered by some other principle in a different way, which does not define the fundamental laws of finite arithmetic.[20] Thus the system of axioms which define the transfinite aggregates is prior to the system which defines the finite numbers (in metaphysical language: "finite things are limitations of infinite," ἀόριστον πρὶν ὁρισθῆναι); that is to say: the actual infinite is presupposed by the potential infinity.

Infinity of Series. If the subjective ability to continue a certain kind of operation *in indefinitum* does not belong to the constructive value of infinity, then what is the meaning of this peculiar term? It has to be determined without any reference to subjectivity—otherwise it would have no meaning at all. Let us start with the consideration of infinite series.

We may express the approximate value of π in decimal notation.

$$\pi = 3.14159\ldots$$

mind for the purpose to grasp the infinity *in actu*. The same might be said against Kant's doctrine.

[20] Dedekind, *Was sind und was sollen die Zahlen?* p. 37.

It is plain there is no sense in speaking forthwith about the whole decimal series in this expression, simply because the terms of the series are not represented by it. The equation gives no enunciation of what numbers are to be united into a whole; however long we may continue the enumeration of decimal terms, the value of π still remains under the curse of the undeveloped "potentiality." Nevertheless, π has an exact geometrical value, being a symbolic expression of a certain type of relation which cannot be expressed by any other value.[21] Otherwise, *every* decimal in the above series not only can be but objectively *is* determined — whether we compute it or not—by a certain method of operation, *each* of them *is* a result of exactly defined proportions and relations between the finite numbers. The theorem of Taylor supplied the foundation upon which have been based different methods for estimation of the value of π. Suppose we have carried out the calculation of π until we have reached the 707th decimal; we ask: are the 708th or 1000th decimals objectively undetermined? Would they be "created" in the process of our further calculation and begin to be only in that moment when we *know* their value? However it may be with the question of the dependence or independence of being upon consciousness, it is evident enough that the logical value of a certain mathematical proposition does not begin to "exist" with our temporal act of knowing it; all the roots of a certain algebraic equation of *n*th degree, for example, have their mathematical "existence," i. e., they are perfectly determined, in spite of the fact that we are forever unable to know them. How much more right then we have to conclude that *every* decimal in the objective value of π is determined by a certain type of preserial relations (theo-

[21] Comp. Couturat, *De l'infinie mathématique*, pp. 216-217: "En admettant que le symbole $m/0$ n'ait pas de sens numérique, il ne laisse pas d'en avoir un parfaitement intelligible en géométrie: car ce qui est absurde ou illégitime au point de vue du nombre pur ne l'est plus au point de vue géométrique." Comp. pp. 257; 213-256; 488-503.

rem of "middle worth"); and we are able to continue our calculation indefinitely only *because* every term—whether we actually know it or not—is objectively determined. Not in the process, but in the method of this determination our series becomes infinite. We should have no right to speak of the infinity (not even the potential infinity) of our series were there no *law* generating all the terms of the series. Therefore the working principle and real essence of infiniteness in our present case is represented by this generating law which renders it actual. Mathematically the infinite series may be regarded as given in its totality ("the series defines an infinite number")[22] only when its "general term" is given, because the constructive law is then expressed immediately in the form of series:

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\cdots \pm \frac{1}{2n-1} \pm \cdots$$

Of course the infinity in this case would be meaningless if we tried to construct it by the successive addition of the terms; because this addition can never be fulfilled and the series does not mean anything else but the virtual determination of "every" term by a certain law; and the indefiniteness of series is nothing but a reflex of the actual infinity of law, being a negative expression of its positive character. Thus the infinity, being an instrument or methodical concept rather than one of process, is free of every continuation; it cannot be constructed by the methodical means of continuation, because the process as such is irrelevant and does not belong to the purport of infinity; it is rather an expression of the structure of certain processes than a concrete process by itself.

The reproach that we are unable to accomplish the measurement of a circle whose diameter is equal to one, does not prove the impossibility of such a circle; its circum-

[22] This terminus: infinite number is introduced by Dingler to denominate every mathematical expression, containing infinite chain of operations and united by a certain and expressible law.

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ference possesses its exactly determined value *in the constructive law* of π; this transcendent value is *not approximately but exactly* determined by this law, expressed by its "general term" or more deeply by Taylor's general formula. We don't need to estimate all the terms for the purpose of determination of series, which is sufficiently determined independently upon the estimation of all approximative value in the series.[28] It is absolutely wrong to suppose that the mathematical determination is possible only by the arithmetical calculus; it can be afforded also by the simple indication of the constructive law. The arithmetical determination represents only a particular case of this general rule, every "number" is but a symbolic expression of a certain "type of order"); it is not a self-contained entity separated from the concrete reality but rather a peculiar way of bringing order into the concrete world of experience, a method of operation. The mathematical "existence" of the various kinds of numbers signifies only the possibility of determining magnitudes by their constructive law; "existence" means nothing but determinateness of whatever sort. The number 2 is not less determined when not defined by the constructive serial law (general term)

$$2=\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}+\cdots$$

than by its arithmetical definition:

$$2+1-1$$

Therefore the number 2 may be an "infinite number" as well as π; the difference is only a formal one, the determination of irrational number by infinite series may be used as their definition. This is impossible in reference to the rationals only, because it would be an apparent circle

[28] Comp. Euler, *Vollständige Anleitung zur Algebra*, Ges. Werke, Vol. I, pp. 50-51. He says: "One cannot say, that $\sqrt{12}$ is in itself undetermined, but from what has been just said it follows only that $\sqrt{12}$ cannot be expressed *by fractions*, nevertheless it necessarily has a determined value."

in definition as may be seen from the equation given above. Returning to our previous example, π is sufficiently distinguished from every other value by the simple character of its constructive law and can be used as a well-defined complex of numerical relations;[24] its value is an actual one, and we have the right to regard it as actual only because it can be expressed in the terms of series having on infinite law. Thus *infinity is rather a property of law than of series as such.*

As to the character of the constructive law in general, another limitation has to be made. It is evident that our reasoning has no application to the divergent series. Of course, the law of a divergent series may be actual too, but it is meaningless to call it constructive, because it does not construct anything. The infiniteness in this case loses its purpose and that may be so important as to lose ground. For infiniteness is completely expressed only in and by the fulfilment of a certain task; where no determinate task is set, no fulfilment is possible. This might have made the concept of "limit" of such a great logical importance. But, from the logical point of view the limit is a statement of problem rather than a solution of it; the "existence" of limit is proved by the constructive law of series, not contrariwise, and the peculiar "jump" made by our mind in transition to the limit remains still unexplained and unintelligible if the methodical sense of actual infinity is disregarded. The infinite constructive law leads us to a determinate, mathematically positive result, i. e., it justifies the establishment of limit only when the series satisfies the conditions of convergence; that is to say: "the *infinitum* is possible and intelligible only in its relation to finite magnitudes, in its logical activity as constructing and pro-

[24] Every "infinite number," like π, may be regarded as an instance of such a "whole" which contains more than *all* its "parts"; because every "part" or "cut" of the series is a rational number (according to the definition) and the "whole" leaves the limits of rationality in principle, breaking the methodical law of it.

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ducing the finite result" (metaphysically: "to be principle is essential for the ἄπειρον," "*das Prinzip erscheint nothwendig*").

The following important conclusions may be derived from the analysis of series given above: (1) Infinity has its logical basis in the constructive law (positive) which *makes* the series endless (negative). (2) The possibility of continuing a certain process indefinitely is nothing but a reflex of the objective activity of certain laws (not every methodical law gives this possibility). (3) The purport of infinity is justified only if it gives and implies a method for the genetic creation of the finite (conditions of convergence). (4) The qualitative moment in the logical explication of infinity turns out to be the methodical efficiency and defines itself more precisely as the logical activity of a certain type of laws.

The infinitely large (transfinite) numbers.[25] We are brought to the same conclusions by the consideration of transfinite numbers. What did Cantor mean by his "actual infinity"?

If we compare the whole lot of algebraic numbers with the series of integers, no conclusions can be immediately derived as to the quantitative difference between the two assemblages; there is no sense in speaking of the totality of an indefinite fraction, in default of a method for its construction. To Cantor chiefly is due the credit of preparing the way for the methodical comparison and mathematical treatment of such indefinitely large multitudes. Instead of classifying "numbers" by themselves he classifies the roots of *all* the algebraic equations, and in doing so he makes it possible to insert them in a proper order in which every equation finds its "enumerable" place in accordance

[25] This paragraph I suppose to be in agreement with Pr. Brown in his remarkable article: "Intelligence and Mathematics" (*Creative Intelligence*) in which is emphasized the methodical conception of transfinite numbers. But I cannot agree with the author concerning the potential character of this type of "numbers."

with its "height"; the classification of real roots follows immeditely and of itself because every equation has a finite number of roots which may be disposed in a proper order in accordance with their magnitudes. By this simple method of disposition he was able to coordinate all roots and consequently all algebraic numbers to the series of integers; obviously in the process of this coordination, not one root remains without a "number."[26] This method of "univocal coordination" first opens to us the logical possibility of applying the concept of "whole" to such indefinite aggregates, "wholeness" (infinity) signifying nothing but the unrestricted action of a certain law of coordination within certain limits. The aggregate of all the algebraic numbers can be regarded as a "whole" (transfinite number) only because *it can be arranged in the same way as the series of integers is arranged*; this prior constructive principle makes both classes of numbers equivalent, in spite of their indefiniteness rendering them of the same "class" and endowing them with the same "power." Every different "power," i. e., every *aleph,* means a different way or arrangement rather than a new (larger) quantity; by the general method of "covering" (*Belegung*) we are able to produce new and newer kinds of infinity, i. e., ever new ways of arrangement. Thus here also infinity is perfectly imprisoned in the finite magnitudes and expresses nothing but a peculiar method of organization of our usual experience.

The mysterious equivalence between "whole" and "part" loses its paradoxical character if we consider the situation from this methodical point of view. *It does not express equivalence in* quantis, *but only an equivalence in* methodis. The whole lot of algebraic numbers quantitatively is the same aggregate as the whole lot of integers, but differ-

[26] Cantor, "Ueber eine Eigenschaft aller realen algebraischen Zahlen (*Journal für reine und angew. Mathematik,* Vol. 77, pp. 258-268).

ently arranged; the "addition" of new terms does not change anything in this arrangement, which *methodically* remains the same, i. e., does not increase it at all, just as the designation of new officials does not increase the population. Cantor categorically distinguishes the "logical function" by which the transfinite numbers are established and proved, from the method of successive addition of terms.[27] The transfinite number, consequently, does not "consist" of its parts, because it is not constructed by regular addition. To take away a proper part from a certain transfinite aggregate does not change anything in it, *because the part has never been added.*

The infinitely small. The usual protest of mathematicians against the actual character of infinitely small elements has its ground in the tendency to understand it in terms of extensive magnitudes, i. e., infinitely small *parts.* Of course under such an aspect the concept of the differential becomes impossible and even contradictory. But still the infinitely small may be regarded as actual from the standpoint of such categories as lie beyond the competency of the primitive opposition between part and whole; this opposition may be irrelevant in the process of its logical construction. For differentiation is not division, and the mathematical procedure of this operation has no resemblance to the process of division. It is logically impossible to establish the concept of an indivisible part.[28]

The task of differentiation according to Leibniz is not to find infinitely small parts; the differentiation has to do with laws, i. e., functions, instead of with extensive magnitudes. If the constructive law of a certain line is given, we determine by differentiation, not a "point," but the direction of the tangent in this point; if a certain law

[27] Cantor: *Grundlage einer allgemeinen Mannigfaltigkeitslehre,* No. 11.

[28] Leibniz, *Mat.* III, 524: "Etsi enim concedam, nullam esse portionem materiae quae non actu sit secta, non tamen ideo devenitur ad elementa insectabilia aut ad minores portiones, imo nec ad infinite parvas."

of movement is given, differentiation determines the velocity at every moment of this process. Thus differentiation is not an algebraic action with extensive magnitudes but a way of dealing with laws or functions as such: when a certain law is given, differentiation permits us to determine the action of this law at every moment and for every element, i. e., to find the derivative function. On the contrary, if the derivative function is given, i. e., the action of a certain law at every moment is known, we are able by the inverse operation of integration to determine the law itself (the equation of a certain curve for instance), i. e., to find the original function (*data lege declivitatum curvae, posse describere curvam*). Within this operation the curve is regarded not as an aggregate of points, not as what it "consists of," but as a continuous change of direction; in the same way we are to conceive movement not as a product of "rests" but as a continuous change of velocity. The well-known paradox of Zeno is a simple case of fallacy; from the fact that in the o of time a point traverses no space, nothing follows as to the point's velocity. The expression o/o is arithmetically undetermined and undeterminable; but Zeno fallaciously ascribes to this point a determined velocity—rest being a peculiar case of velocity, where $v = 0$.

The differential does not exist as a part of the integral; under the aspect of part and whole (arithmetically) it is a pure nothingness; it has a meaning only as an instrument for certain operations with magnitudes, not with numbers. From this numerical point of view o is an expression for the simple negation, or absence of being: o/o here has no sense and must be regarded as an absolutely undetermined form, precluding all mathematical treatment. In the series of magnitudes the zeroes do not exist at all; here the concept of zero must be supplanted by the concept of "moment" which always conserves its specific qualitative

character. A point in space, an instant of time and a moment in a movement are qualitatively distinguished in spite of the absence of any quantitative element in them. But the concept of this peculiar quality again is a statement of the problem rather than a solution of it; the quality has no mathematical expression and cannot be used in the process of calculation. What may be regarded as conserved is not the quality but an exactly determinable relation which is active in every moment of the process. In this sense the point of a circle is different from the point on a parabola, because each keeps the direction continually produced by all other points of its "class"; every element of a curve implies the law of its curvature; every moment of the movement continuously keeps the law of its velocity. Therefore the point, in so far as it is an element in the continuous change of direction, is not a simple null, but such a null as logically to imply the law of the whole line, i. e., the "infinitely small element" of "differential."[29] This infinitely small element has a meaning only in reference to a corresponding magnitude, determined in its whole character by a determinate formula or law (*interventu infiniti finitum determinatur*).[30] Another account of the infinitely small is possible. According to Leibniz's principles the differential has no positive meaning in itself; it seems to be only a symbolical expression of what is logically tctive *behind* it in the methodical meaning of the derivative function ($dx:dy$).[31] Is it possible to ascribe any positive value to dx independently of dy? Veronese tries to solve this problem geometrically. He defines the infinitely small "segment" of the order m as follows: "If a number in relation to another number is infinitely large, then this second number in relation to the first may be called infinitely

[29] Leibniz, *Opera Mathem.*, IV, 218: "Interea infinite parva concipimus non ut nihila simpliciter, sed ut nihila respectiva....id est ut evanescentia quidem in nihilum, retinentia tamen characterem ejus quod evanescet."

[30] *Ibid.*, VII, 53. [31] Comp. also Euler, *De calculo integrali*, I, 2.

small."[82] In a special theorem he emphasizes the actual character of elements so defined, exactly distinguishing them from the "indefinitely small" elements: the latter still remain in the process of diminution and belong to the series of finite magnitudes; but it would be meaningless to seek the infinitely small segments in the series.[83] Bernoulli already believed the actual existence of infinitely small elements: "Sic omnes termini hujus progressionis, ½, ¼, ⅛, 1/16,.... actu existunt, ergo existit infinitesimus....si decem sunt termini existit utique decimus, si centum sunt termini, existit utique centesimus...ergo si numero infinito sunt termini, existit infinitesimus." But Bernoulli still seems to believe that the *terminus infinitesimus* is given in the series itself; of course, were this true, Veronese says, the concept of the infinitely small would involve a contradiction, because all the terms of this series according to the definition are finite. But it is logically possible to regard the series as defining a certain element beyond itself, which does not belong to the class of numbers given in the series, being always smaller than every term of it; and nevertheless this element may have some exactly determined and definable properties. In a certain circle of problems the assumption of such element may even be inevitable. Suppose we hypothetically accept the assumption of such a system, where "every finite segment, variable as to length and becoming indefinitely small, contains an element, which is different from its terminal points"; in the first place this presupposition is logically possible and implies the definition of infinitely small elements given above; in the second place it is obvious that certain systems (for instance the system of the spatial points) satisfy this hypothetically accepted condition.

From what has just been said it seems to follow that

[82] Veronese, *Grundzüge der Geometrie von mehreren Dimensionen*, p. 116.
[83] *Ibid.*, p. 141.

the infinitely small may have mathematical existence by itself. But still it remains true that all the determinations ascribed to this element are, properly speaking, of a derivative nature. What Veronese might have had in mind is only a convenient way of expressing certain properties and proportions in a certain class of systems called continuous. He explains himself clearly in this way.[34] If the distance between the two foci of the ellipse remains only indefinitely small, without any suggestion of an element of a different order transcending the potential series of such indefinitely small distances, the circle might never be considered as a particular case of ellipse. The "actual existence" of an infinitely small distance in this case means nothing but the possibility of passing from the formula given for the ellipse to that for the circle. The "actual existence" has here no meaning beyond the methodical value of certain operations; and this methodical value of the "infinitely small" element consists in what it is doing in the system, rather than in what it is. The correlation between the finite distance and the infinitely small element defined by the process of its continuous determination, is no relation between quantities, but in our present case, the relation of affinity between two different laws (circle—ellipse). Their "truth" consists in what stands behind their formal definition, in the methodological back-ground of their "existence." Still in terms of our present example, we may say that the point (as infinitely small distance) can be regarded as a "part" of the line only because a certain class of analytic forms (circles) can be regarded as a "part" of another more general class of forms (ellipses). It is obvious that the problem here again harks back to the problem of qualitative infinity.

Résumé. From what has been said it follows that infinity in all the cases of its application has a purely method-

[34] *Ibid.*, p. 144.

ical value. It is not a "thing in itself," not something ready given and self-existent independent from science; it is not a "thing" of whatever sort; it is a method, rather a methodical aspect of reality than reality itself. I don't want to allege that it is a pure product of our mind, unless we understand this term "mind" in a purely logical sense, as a system of methodical presuppositions of science, action or art. Then and only then, in this exactly restricted sense of "our mind," it may be logically created by it, i. e., every instance of infinity may be and, as a matter of fact, is a result of certain presuppositions. The reproach of artificiality does not affect our position in any way; in this broad and vague sense everything may be called artificial; I don't see any reason why any finite magnitude or any limited field of experience is less artificial than a transfinite number. We are too much inclined to forget, that a long period first of biological adaptation and then of logical and mathematical reasoning were required to perceive the limits of the real objects and to conceive the meaning of the "end." It is an old truth that all the boundaries in this world are artificial, i. e., they are based upon a long system of presuppositions. But since these presuppositions are not artificial at all, since they have their meaning and purpose, the result of their logical activity loses its artificial character also. Thus to persist in the thesis that everything in our world of experience is limited, is in itself a logical limitedness: It must be considered as a modern positivistic extreme, as a reaction against the metaphysical exaggeration of the value of infinity. The opposition of finite and infinite is not a contraposition of the different realms or worlds separated from each other; it is only a cooperation of two different methods one of which is quite as justified as the other.

HENRY LANZ.

PALO ALTO, CALIFORNIA.

IMAGINATION.

SERVANT OR MASTER.

REASON'S eye is calm and steady,
Gazing ever straight ahead,
Seeing clearly every object
In its level vision spread.
But Imagination cries: "Look upward!
Here are wondrous things to see!
Leave your sober, steady plodding,
Trust my wings and fly with me."
Reason answers: "I will follow
Throughout all your fairy land,
But forget not, pretty maiden,
I shall always hold your hand."
Then the sprite Imagination
Guides him to the Ivory Door,
Lets him see the deeper meaning
Of his slowly gathered lore.
Never master had a servant
Who could give him such delight,
But 'tis well that Reason watch her,
See her safely home at night.

The scholar struggles slowly
Through the records of the past,
Sifting, balancing, rejecting,

Pondering o'er their meaning vast.
Suddenly Imagination
Breaks from Reason's curbing rein
As the lightning leaps from heaven,
Flashing through the startled brain
Swiftly vivid pictures, blending
In *one truth* the scattered train
Of the facts which toil unending
Strove to reconcile in vain.

He who walks beside the river
Hears its vexed and sullen roar,
Sees it sweeping swiftly onward,
Sees—a fact—and nothing more.
He who views it from the mountain
Sees a gleaming silver rod,
Silent, motionless, completed,
Like the changeless truth of God.

There's a pathway up the mountain,
Steep, laborious, and slow,
Lighted only by the witch-fire
Of Imagination's glow.
That lone path which thought has traveled
Since the Reason's earliest youth,
Struggling upward toward the cloud-cap
That still veils the Greater Truth.
Not for fame and not for riches
The explorer scales these heights,
But for the exhilaration
Of revealing hidden lights.
There's no joy for human nature
Like the mind's exultant thrill
When the new-born thought leaps living,
Bringing that ecstatic chill

Which has in it more than nature,
Holds the heart and brain in thrall,
Makes us wonder, spite of reason
If we're not immortals all.

When Galileo saw the lamp
Swing slowly to and fro,
A light leaped up within him,
'Twas Imagination's glow.
His reason fed and fanned it
Till its radiance burned away
A dozen dogmas of that church
In which he came to pray.
When Newton saw the apple fall
Imagination gleamed.
With all his hoarded learning
He never yet had dreamed
Of what that searchlight showed him
Which his reason gripped and steered
Through vast sidereal spaces
Where worlds on worlds are veered.

When the thinker meets the barrier
Of the "Ultimate First Cause,"
Reason fails him, for the problem
Seems transcending Reason's laws.
Then Imagination murmurs,
"Set me free and I will tell
All that Reason cannot show you,
All the truths of heaven and hell."
When the seeker, worn and weary,
Meets no answer to his quest,
Finds his Reason baffled, beaten,
Helpless at his great behest,
Yearns to know—what mortal knows not—

That which follows after death,
Then Imagination whispers,
"Lean on me, for I am Faith."

But if once Imagination
Is set free from Reason's hand
She assumes a thousand figures,
For they're all at her command.
Now an angel in the brothel,
Now a devil at the shrine,
She endows each human error
With an origin divine.
She has led Utopian dreamers
Into many a grave mistake,
And inspired the grim fanatic
To burn Reason at the stake.
Like the "Genius of the Bottle"
In the oriental tale,
She's a servant true and mighty
Till the magic word shall fail,
Then she becomes the Master,—
Oft a tyrant and a curse,
Leading blinded Reason captive,
Speeding on from bad to worse,
Till at last the frenzied dreamer
Thinks he hears the voice of God
In his wild Imagination,
Uncontrolled by Reason's rod.

All the palsying superstitions
That in ignorant minds find place,
All the cruel, false "religions"
That have cursed the human race,—
All the torments and the furies
That have harried every land—

Are Imagination's children
When released from Reason's Hand.

Yet the greatest truths discovered
Own Imagination's sway,
She the seeress, she the wakener,
Lights the torch for Reason's way.
All of poetry and music,—
All the beauty and the grace
Of the arts that help to sweeten
And uplift the human race—
Are Imagination's children,
Owning Reason as their Sire,
Sane and splendid, looking upward
With the soul's divine desire,
Yet they are the true half-brothers
Of that deadly bastard spawn
Which has kept the shadow lingering
O'er the promise of the dawn.

C. L. MARSH.

THE SUPER-SOUL.

FROM Sun-begotten sires of earliest life,
We draw our being, stronger through their strife.
They fought their way from formless germ to man,
With "climbing instinct," following nature's plan—
The plan, however planned—that life shall rise.
The "Right" of Nature lives, the "Wrong" still dies,
Not individual, but of the race.
The greatest man leaves no enduring trace
Save in the building, strengthening of the power
From which the coming man derives a dower
Of added strength and upward striving will,
That cumulates through generations still.
Not power of wealth or rank, but that of thought,
Which lives and grows forever and has wrought
All that has lifted life from brute to man
Since first the gleam of reason's light began.
Thought moves forever on in peace or wrath
Though blood of retribution stain its path.

Science, with searchlight of unbiased will,
Through dust of dream and dogma peering still,
Strikes to the solid rock of Nature's youth
And says with confidence: "Here rests the truth;
This earth was once impossible of life,
Now with life's varied forms the earth is rife."
The necessary sequence checks the breath.

UMI

By unknown forces *Life was born from Death*;
By natural law, to chemist still unknown,
Man has evolved from inorganic stone.
Strange confirmation of primeval thought—
From dust of earth the Gods mankind have wrought.

But whence is this that lifts us far above
The dawn of life, this power of thought and love?
This upward striving, this "divine desire,"
This restless star-ideal, ever higher?
The mind that through the tiny cells of brain
Can weigh the universe, and call again
The wisdom of all ages to its side,—
The heart that still with patience can abide
The scorn of *men* for higher love of *man*,—
The souls creative, lifting us to scan
The half-veiled beauty which we dimly see
In those rare hours that seem from earth set free—
Was this potential infinite alone
In that first germ that lived from lifeless stone?
Or was there influx of a mighty soul,
In-forming, lifting toward a rising goal—
Through each new form infusing greater strength
And greater self-dependence, till at length
The soul of conqueror Man, still incomplete,
Sees all the powers of nature at its feet?

Yet life evolved from inorganic stone,
How—matters little, so the fact be known.
Then was the power to live before the life,
'Twas in the stone itself through all the strife
Of world-formations; in the glowing mold
Of lifeless suns, in sunless realms of cold.
From unbeginning past we reach to-day;
To endless future lies our upward way.

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In that eternity from whence we came
Lives still the potence of the higher aim,
The concepts *Good* and *Beautiful* and *True*
Forever widen to man's widening view.
There is no absolute, no perfect whole.
Perfection means stagnation, *growth is soul.*
The mountain tops that genius may attain
Rise ever higher from the grovelling plain,
But o'er their loftiest crags still loom afar
Unending peaks beyond the utmost star.
The immortal past is parcel of our life,
It breathes into our souls the upward strife.
The scientists, the poets and the seers,
The best and wisest of the bygone years,
Point ever upward from the heights they won,
And cry: "Beyond! Above! 'Tis but begun!"

Perhaps correlative to high desire
The Universal Soul's undying fire,
The *Growth-Law which is God,* informs us still.
Unrecognized, dream-felt, it molds the will;
It beckons ever to remoter goal,
The all-embracing Mind, the *Super-Soul.*

C. L. MARSH.

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THE TWO-HUNDREDTH ANNIVERSARY OF THE BIRTH OF WINCKELMANN.

THE year 1917 is not only the four-hundredth anniversary of the birth of the Reformation, but also the two-hundredth of the birth of Winckelmann, the founder of scientific archeology and the father of modern art criticism. There is more of similarity in the work of Luther and Winckelmann, if both are judged by the influence which they wrought on posterity, than appears at first sight. While the one brought a complete change into the attitude of men's minds toward religion, teaching that an independent judgment is the inalienable right of every religious man, the other effected no less complete a change in the world of esthetics, by overthrowing the false taste in art and wrong conception of classical learning which obtained throughout Europe in his day, and by laying the foundations of a wholly new science.

In reading the biography of Winckelmann by Karl Justi[1] one feels that he is in the presence not only of one of the greatest scholars, but one of the greatest of men. His greatness as a scholar is indubitably attested by the scientific work which he left behind him, as well as by the influence which he exerted not only over his immediate contemporaries, but over the whole world of learning and culture since; his greatness as a man is no less clearly discernible in the infinite capacity which he possessed for

[1] *Winckelmann, sein Leben, seine Werke und seine Zeitgenossen,* 3 vols. Leipsic, 1865-72. 2d ed. 1898.

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overcoming the almost insuperable difficulties of his early career until he reached his life's ambition. Nor was he only concerned with books and monuments, but with men, constantly seeking the help and inspiration of true friends, since he believed that friendship was the greatest of human virtues. For one born and schooled in adversity in an age and in an environment whose ideals were out of harmony with his very nature; for one who not only lacked the means to properly prosecute his studies, but the inspiration of contemporary science and art; for one who had never seen a genuine monument of ancient art until he had passed his thirtieth year; for such a one to have raised himself by sheer ability and industry to the highest place in European scholarship and to have been the means of completely reversing the attitude of his day toward art—all this discloses greatness of a rare order. For Winckelmann was not one of those fortunate mortals who are born in the lap of luxury, whose genius is slowly but easily unfolded throughout a long life and at the end crowned with great rewards; on the contrary he was of lowly birth and only with incredible difficulty accomplished his life work, and then was suddenly cut off by an appalling calamity after having barely passed his fiftieth year. His brief life was one of great contrasts in which the shadows and lights were about equally balanced—his journey to Rome in his thirty-eighth year dividing it into two distinct parts. It was the contrast of want and competence, of removal from the rudest environment to association with the world's best collections of art and intercourse with the greatest personalities of Italy and Europe, of being the teacher of recalcitrant village school children to becoming the preceptor of Europe and posterity. It is surely a life story well worthy our study and emulation.

To understand the character and significance of the change in the esthetic view-point wrought by Winckel-

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mann's influence, we must understand how it was that Italian taste with its prejudice in favor of Latin studies over Greek and indifference to the latter had dominated Europe for two hundred years before his time.

The study of Greek, which had been so enthusiastically begun by the Greek immigrants and Italian humanists in the fourteenth and fifteenth centuries, as well as the great period of Italian art beginning with 1300 and so intimately connected with the commercial prosperity of the free states of central and northern Italy, began to languish after the first quarter of the sixteenth century. This decline was primarily due to the loss of political independence in these states during the disasters which befell them in the time of Michelangelo. Italy, the fairest and richest of countries, then became the prey of foreign armies and could no longer under the leadership of the popes present a united front against invasion. An army of Charles V sacked the eternal city in 1527 and took Pope Clement prisoner; two years later Florence was besieged by another imperial army and by its surrender in 1530 lost its liberty, and by the reestablishment of the Medici in 1532 as hereditary dukes of the capital and later of all Tuscany, Italian freedom was doomed. From then on until 1796—over two hundred and fifty years—Italy had no political history of its own: its annals were filled with records of dynastic changes and redistributions of territories, and it became the theater of desolating wars fought for the most part by the armies of contending foreign princes and for ambitions in which the Italian people had no share. The brilliant aristocracies which had long cultivated humanistic studies were ruined and the predominant influence of the reformed Catholic Church looked with no friendly eye on the worship of pagan ideals, an attitude which was bound to divert Italy from classical learning. The Greek elements and influences in Roman art and letters had been so thor-

oughly assimilated at the end of antiquity by the Imperial Age of Rome that there were few Italians in the seventeenth and eighteenth centuries who were aware of their independent Greek origin. The Mohammedans were holding Greek lands in thraldom, and no one visited them to bring back a truer knowledge to counteract the growing tendency to treat Roman studies as superior to Greek and to look upon them as original. Patriotism, moreover, naturally led Italian scholars to exalt their own country as the center of the old Empire of Rome. They knew Italy's debt to the Romans in both literature and art and enthusiastically imitated them without any critical idea that the Romans had largely copied the Greeks. The fact that Italian was descended from the language of Rome made it easy for them to unlock the treasures of Latin literature. Thus, generally speaking, it had come to be customary in Italy to ignore Greek studies and to prefer everything Roman, and this way of looking at things spread over all Europe until finally, in Winckelmann's century, Italian taste, founded upon a wholly mistaken historical conception, ruled all cultivated nations. The great Italian humanist, Julius Cæsar Scaliger, long before had declaimed against Greek in favor of Latin, and his book of Latin verses — the *Poetice* — which appeared posthumously in 1561, remained a standard of taste down into the eighteenth century. Though the French historian de Thou exalted him above all scholars ancient and modern for his learning and talent, we know that he only looked upon classical studies as an agreable relaxation from the severer pursuits of life. In fact his chief amusement in later years was the composition of Latin verses. Thus within a century of Byzantium's fall, the Renaissance had already begun to take on in Italy its characteristic Roman bias.

In France the sixteenth-century Greek tradition inaugurated by Stephanus and Turnebus soon began to wane.

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The French schools were deserted by Joseph Scaliger in 1593, by Casaubon in 1610 and by Salmasius in 1631. By the end of the seventeenth century classical enthusiasm had yielded to a taste which found pleasure in ridiculing Greek studies with characteristic Gallic wit. The age of Louis XIV,—the founder in 1663 of the Academy of Inscriptions—was marked in the years 1687-92 by the literary quarrel between Perrault and Boileau over the respective literary merits of the ancients and moderns. In his *Parallèle des anciens et des modernes,* Perrault, after a superficial survey of ancient and modern literature, gave the palm to the moderns. He declaimed not so much against the genius of the ancients as against their technique, the impersonal and objective character of their art. He compared Homer's immortal lays with the ballads of the Parisian street singers and looked upon his heroes as of lower stature than the dandies of Versailles, more like their landed thralls. His book was the signal to a controversy which passed over to England and again, in the days of Antoine Houdart de la Motte and Fénelon, returned to the land of its origin. La Motte, like his master, was an enemy of Greek and measured Homer by the rules of romantic French poetry. Voltaire, the dates of whose long life included those of Winckelmann's, expressed his sorrow that "the most beautiful language of the world" was neglected in France in his day. While praising the truth to nature and the descriptive power of Homer, he nevertheless found many unhewn stones in his marble palace and was content to set the second, fourth and sixth books of the Aeneid above not only the Iliad but all Greek poetry. He thought that the *Jerusalem Delivered* was at least the equal of the Iliad. He admired the dignity of Demosthenes, but looked upon the immortal Aristophanes as a mere *farçeur.* Plato did not please him because he made virtue too attractive and vice too repulsive; in his opinion Cicero was the equal

of any Greek thinker. Out of respect for his judgment of the Iliad we must remember that Voltaire also essayed to write an Epic, which, I fear, but few even of the professors of French to-day read with any pleasure.

In England humanism had not yet recovered from the effects of the Civil War. There was no great name in classical scholarship until that of Richard Bentley, who was destined to become the greatest figure in the learned world of Europe during the first half of the eighteenth century. Sir William Temple, who knew no Greek, nevertheless entered into the controversy begun across the channel and championed the ancients with his *Essay on Ancient and Modern Learning*. A challenge to prolong the conflict was given by his statement that the best examples of Greek literature were the fables of Æsop and the letters of Phalaris, which he looked upon as nearly contemporary. The challenge was first accepted by Wotton, who, in his *Reflections upon Ancient and Modern Learning* (1694), calmly examined Sir William's reasoning. His friend Bentley told him that the fables of Æsop were not the work of Æsop at all and that the letters of Phalaris were a late forgery, the work, perhaps, of a sophist of the second century A. D. Temple's advertisement made a great demand for these worthless letters, and a young Oxford scholar named Boyle published an edition in 1695 . A second edition of Wotton's essay was followed in 1697 by Bentley's famous *Dissertation on Æsop and Phalaris*. Nothing can better show the real state of Greek studies in England at his time than the fact that for some time public opinion favored the enemies of Bentley; however, the second edition of his *Dissertation* in 1699 marked a new epoch in English scholarship by heralding a new era of criticism. We have interesting hints of how Greek was neglected at Oxford at this time in Macaulay's *Essay on Addison*. While Addison had an intimate knowledge of the Latin poets

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and could write an excellent Latin style, his knowledge of Greek, though such as was deemed respectable at Oxford in his day, was evidently less than that which is carried away by many high school boys of to-day. An account of his Italian journey also shows how preponderating was his interest in things Roman.

In Winckelmann's own land classical studies had fared but little better. Their systematic study inaugurated in the fifteenth century by Huysman and continued by the labors of the humanists Reuchlin, Melanchthon and Camerarius had already begun to languish by the close of the sixteenth century. The leaders of the Protestant Reformation, Luther, Melanchthon and Zwingli, were all classically trained men whose minds had been broadened and whose powers of expression had been increased by the study of the Latin and Greek classics. Most of the Latin schools of the sixteenth century were founded under the direction of Melanchthon, and his educational plan was taken over by the universities which he reorganized. In his *Discourse on Reforming the Studies of Youth*, which he, a youth of twenty-one years, delivered as his inaugural as the first professor of Greek at Wittenberg, Melanchthon expressed his determination to plead the cause of the classics against those who found them "more difficult than useful" and who maintained that "Greek was studied only by disordered intellects and that, too, for display." His appointment at Wittenberg marked an epoch in German university education; for under this *praeceptor Germaniae* Wittenberg became the school of the whole nation. In laying aside the old scholastic methods of instruction, he showed that he had caught the real spirit of the Renaissance and was fitted to be one of its greatest leaders. In lecturing on Homer he announced that he, "like Solomon, was seeking Tyrian brass and gems for the adornment of God's temple" and he also asserted that "by going to the sources we are

led to Christ." But despite its glorious initial promise the Reformation was bound to react detrimentally on classical learning. Luther, though he began his work at Wittenberg with lectures on Aristotle's *Physics* and *Dialectics,* soon found his influence harmful to the new theology and came to look upon Aristotle as the personification of scholasticism, the great enemy of the Church. He, therefore, wished to banish the *Ethics* and *Metaphysics* entirely from the university curriculum and to retain only the *Rhetoric, Poetic* and *Logic,* because these works might help young men to preach and pray better. The whole Protestant principle in art, isolated by the cleavage from Italian influences, was destined to cut Germany off from the ancient tradition of beauty and culture. The Thirty Years War in the following century had, like the Civil War in England, a disastrous effect upon every form of learning and culture. With the peace of Westphalia in 1648 neither art nor classical learning revived. The age of the giants of humanism had passed. After the death of Camerarius in 1574 there was not a name of importance in German classical scholarship for a hundred and fifty years until that of Johann Albert Fabricius (died 1736) is reached, and he is to be remembered mostly only for his great learning and industry, which won for him the title of the modern Didymus. The Flemish philologist Justus Lipsius had long before heralded the decay of Greek studies by his dictum that Greek was merely an ornament which for a scholar was not an indispensable possession. Latin continued to be taught in Germany and was still largely the medium of university instruction and the language of the learned world. Ancient literature, however, was regarded everywhere as a barren field, quite superfluous to the scholar. In Winckelmann's boyhood Greek was nowhere seriously studied; what Greek was taught was mainly intended for students of divinity for the sake of the New Testament

and the early Church Fathers—that is, as the handmaid of theology. No Greek book of importance had been published for nearly a century and a half, from the time of Sylburg toward the end of the sixteenth century down to that of Ernesti, whose *Memorabilia* of Xenophon appeared in 1737. No Plato had appeared anywhere in Europe since 1602. No Greek text-books, except selections, were to be had. Scientific archeology was yet unknown and scientific philology was yet to be created at Halle by Wolf at the end of the eighteenth century. Porson's gibe that "the Germans in Greek were sadly to seek" was not without point. Only the seeds of the coming revival in Greek studies had been sown. Gesner, the older contemporary of Winckelmann, who was professor of eloquence at Göttingen for twenty-seven years until his death in 1761, was the first to re-introduce the best Greek classics into a German university, by publishing his *Chrestomathia graeca* in 1731 when Winckelmann was a boy of fourteen. This event really marked the advent of the new humanism by rekindling the national enthusiasm for ancient learning. It was Gesner's aim no longer merely to imitate the style of the Latin authors, but to understand the content of both Latin and Greek literature. Though himself a Latinist, he was the first to set a high value on Greek and the first to teach it in Germany and, therefore, may rightly be looked upon as the prophet of the Greek revival to be later instituted by Winckelmann, Lessing and Goethe. The revived classical tradition was carried forward by Ernesti, who, as professor of ancient literature at Leipsic from 1742, was the only official exponent of Greek in any German university in Winckelmann's day; by Reiske, who combined a critical knowledge of Greek with an unrivalled acquaintance with Arabic; and by Heyne, who lectured as Gesner's successor at Göttingen for half a century until his death in 1812. Heyne possessed neither

the enthusiasm nor the penetration of Winckelmann, nor the philosophical nor critical power of Lessing, but he surpassed them both in accuracy and method. Johann Friedrich Christ, the professor of history and poetry at Leipsic after 1754, urged his students not to confine their attention merely to the ancient languages, but to include ancient art, and consequently he may be regarded as the immediate forerunner of Winckelmann in archeology, as Gesner was of Wolf in philology. It is significant of the condition of classical study in Germany in Winckelmann's day that its leading exponents—with the exception of Reiske—were such men as the uncritical Latinists Gesner, Ernesti and Heyne. Many greater German philologists, like Ruhnken and Wyttenbach, had sought the more congenial atmosphere of the Netherlands for their life-work, while others, like Reiske, had been compelled to go there for instruction. Joseph Scaliger, on leaving France at the end of the sixteenth century, had called Holland "the only corner of Europe"; classical scholarship there, which had extended from Erasmus to Grotius, was again flourishing in Winckelmann's time under the influence of the great Hellenist Hemsterhuis, who had founded the only real school of Greek learning which had existed in Europe since the days of Scaliger and Casaubon.

In the last half of the eighteenth century these prejudices in favor of Latin studies over Greek were destined to be overthrown largely by the work of one man—Johann Joachim Winckelmann. Through his influence the older custom of looking upon the relics of antiquity on Italian soil as those exclusively of Roman civilization had to yield to the true origin of these things in Greece. In his first book, *Thoughts on the Imitation of Greek Works in Painting and Sculpture,*[2] which appeared in 1755 just as

[2] The German title of this work is: *Gedanken über die Nachahmung der griechischen Werke in der Malerey und Bildhauerkunst.* 2d ed., 1756.

he was leaving Dresden for Rome, Winckelmann for the first time clearly disclosed the distinction between a Greek original of sculpture and painting and a Roman copy. In the next thirteen years down to his death his researches were destined to revolutionize the esthetic taste of Europe. His notion that there was an independent Greek art, from which Roman art was derived, was, strange as it may seem to us, a revelation to his contemporaries, who had uncritically accepted the interpretations of art works which had been based on the early enthusiasm for Roman history and literature. He showed that the realistic Italian sculpture of the day, which was more interested in anatomical accuracy than in the expression of the beautiful, copied merely the decadent phase of Greek art and that all such dramatic effects were directly opposed to the simplicity and repose of even Roman imitations of Greek works. With the disclosure that Roman art was derivative there was involved a new conception of the general origin of everything else in Roman civilization; for if Roman sculpture, painting and architecture were Greek, it followed that Roman literature and culture in general largely depended upon Greek. The change in viewpoint was to be fundamental and permanent; an entirely new inspiration was to come to Europe—an inspiration only comparable with that of the Renaissance itself. The taste of the succeeding period became Hellenic rather than Roman. Everything Greek—art, literature, history—began to be studied. The resulting intensity and expansion of interest in things Greek we now call the Greek Revival, whose waning we are unfortunately fated to see in our own time. This revival, beginning even in the lifetime of Winckelmann, came to full fruition after his death in the last quarter of the eighteenth century and was destined to become the most prominent spiritual feature of later European history. Lessing, by the publication of his famous essay

Laocoon in 1766—a work chiefly inspired by Winckelmann's ideas and studies—helped the nascent movement by critically establishing the superiority of Homer and thereby lowering the prevailing literary taste inaugurated by the French critics. Goethe's transcendent genius raised it into the higher realm of poetry. But the foundation of it all is to be sought in Winckelmann. He can rightly be called not only the founder of a science—for the principles which he laid down for antiquarian investigation have been followed since with ever increasing results—but also the greatest connoisseur and teacher of the Beautiful. His influence was be no means confined to the world of scholarship. The manifestations of the revival were manifold and far-reaching. The new inspiration entered not only into the more spiritual structure of culture—into the fine arts—but also into politics and every-day life. Here I can only most briefly and generally indicate a few of the more prominent manifestations which resulted from the stimulus of his work.

I have already spoken of the immediate effect of Winckelmann's influence on Lessing and Goethe. It was no less marked on all the Augustan writers of Germany, who owed their greatness to Winckelmann's disclosure of the Greek spirit. The new humanism soon, however, passed the boundaries of Germany and influenced all European letters. Travel to Greek lands began and a long line of English, German, French, Italian, Dutch and Scandinavian scholars studied the monuments on their native soil and wrote glowing accounts of their experiences, which immeasurably enlarged the horizon of scholarship. The new impulse was phenomenal in its influence on architecture, sculpture and painting. The simplicity of form of Greek porticoes and temples caused them everywhere to be copied; the theatrical and sentimental in sculpture yielded to Greek canons of restraint and dignity; Greek simpicity was taken

over into painting. In architecture Schinkel von Klenze and Semper appeared in Germany; Vignon, Hittorff and Chalgrin in France; Soane, Inwood and Wilkins in England, and the architects of many famous Greek buildings in the older cities of the United States. In sculpture the Italian Canova and the Danish Thorwaldsen were followed by the German Dannecker and the English Gibson; in painting the French David, the contemporary of the Revolution and Napoleon, was the best exponent of the new style. Though in all forms of art the imitation of Greek subjects and forms proved ephemeral, the standards of taste taken from Greek art will always remain authoritative. Only after the first quarter of the nineteenth century did the imitation of Greek forms in all the branches of art yield to more independent styles, like the great Gothic revival in architecture, which reached its zenith about 1850, when practically every church built in Europe and America was Gothic. In music the subjects of the operas of Gluck reflected the new spirit. Even in dress and furniture the same spirit was revealed: the short-waisted dress of the Revolutionary period, known as the Directoire in Europe and that of Martha Washington in our country, was merely an effort to recover Greek simplicity: furniture, even clocks, imitated Greek designs. In politics it is hard to overestimate the effect of the revival. The Revolutions in both America and France were certainly largely influenced by the account of republican institutions in *Plutarch's Lives,* the most popular book of the day, while the Greek War of Independence in the last century was due in great part to the sympathy of European scholars and statesmen and men like Byron, who were directly influenced by the sentiments awakened by the second Renaissance of Greek studies.

To have furnished the inspiration and the stimulus for such a change in the spiritual history of the world is indeed

an achievement of the highest order. As Walter Pater[3] has said, the highest that can be said of any critical effort is that "it has given a new sense, that it has laid open a new organ"; and this honor he pays to Winckelmann. Hegel, in his *Lectures on the Philosophy of Art,* has also paid a tribute to the humble German scholar in these words: "Winckelmann, by his contemplation of the ideal works of the ancients received a sort of inspiration, through which he opened a new sense for the study of art. He is to be regarded as one of those who, in the sphere of art, have known how to initiate a new organ for the human spirit." Winckelmann was a man to whom art was both religion and fatherland; when he wrote he thought not of Germany alone nor of his own time, but of all Europe and posterity. When one reflects on what he accomplished and the honor which he brought to his native land, one should not be surprised that his memory has been so highly esteemed in the past by his countrymen as to have amounted almost to Winckelmannolatry, a sort of cult in which he was regarded as a spiritual superman, the patron saint of archeology and art criticism. A more reasonable appreciation of his merits is the custom now long obtaining in Rome and in many of the universities of Germany of repeatedly commemorating his natal day—December ninth—by the publication of contributions to the science which he founded.

It is interesting to know something of the personality and life story of the man who wrought so great a change in men's outlook. Voltaire would hear nothing of the biographies of great writers, for he maintained that the life of a quiet scholar lay open in his works. This is largely true of the authors of scientific works, where facts and methods are the paramount interest and the personality of the writers is secondary. But it is certainly not true

[3] See his essay on "Winckelmann," in his *Studies in the Art and Poetry of the Renaissance,* 1873. I have followed his translation of the passage from Hegel.

of poets, essayists nor of literary men in general, whose life work is more concerned with sentiment and emotion. It is for this reason that we are vastly more interested in the romantic lives of a Cellini or a Shelley than in the more prosaic ones of a Laplace or a Darwin. In the case of Winckelmann, the idea which was the soul of his life's activities was the very human one of beauty and it was through this alone that his personality has influenced successive generations of art lovers. In trying to express this idea he had to pass his early manhood in the uncongenial atmosphere of the north, condemned to subsist by teaching rudiments to children; but he spent his nights in reading Homer and Sophocles, which fired his enthusiasm and finally drew him to the south. The fulfilment of his life's work was of such importance that Lessing, on hearing of his untimely death, could say that Winckelmann was the second writer to whom he gladly would have given some years of his own life—meaning thereby that his life had been shortened by that catastrophe.

Winckelmann was of very lowly origin, the only son of a poor cobbler of Stendal, a town in the ancient Prussian province of Brandenburg. The house in which he was born consisted of only one thatch-roofed room, which was used by the family as working, living and sleeping quarters. His father naturally wished his son to follow his trade and only with the greatest difficulty was persuaded to let the boy go to the town Latin school. Here he received his first instruction from the almost blind rector whose *famulus* he became, reading to him, walking with him and looking after his library. His childish imagination was impressed by the medieval appearance of his native town, by its ancient gabled houses, lofty cathedral and massive city walls and gates, all of which aroused in him thus early a love of the historical and monumental. His boyhood was passed amid great poverty and trials which ever after left their

mark on his melancholy disposition. Years later while viewing the Roman Forum in full emancipation of spirit he said: "One gets spoiled here; but God owed me this, for I suffered too much in my youth." But he who was destined to interpret the charm and beauty of the spirit of Greece to his age, had first to serve an unhappy apprenticeship in the rude intellectual life of Germany. There is no wonder that, as Pater says, after "passing out of that into the happy light of the antique, he had a sense of exhilaration almost physical."

The old rector, seeing the boy's studious nature, wanted him to enter the Church. Consequently it was necessary for him to go beyond the Latin school to prepare himself for the university. At sixteen we find him at the Cologne Gymnasium in Berlin. This was at that time under the direction of a Greek scholar of note, Christian Tobias Damm, the lexicographer of Homer and Pindar. Winckelmann lived in his home as tutor to his children. He soon found, however, that he could get little instruction at the Gymnasium outside of Latin. The recent reform in German schools which had started in Halle under Francke paid little attention to Greek; everything was Latin, German and the positive sciences. Consequently it is no wonder that Winckelmann was more interested in the lectures given at the Academy of Arts and Sciences than in the work of the school. This naturally aroused the hostility of the rector, who showed his resentment by writing in the student register after Winckelmann's name the opinion that he was a *homo vagus et inconstans,* quite unaware on whose side the irony would eventually fall. However, it was not difficult for him to imbue the mind of his young pupil with the idea that Greek was superior to Latin and that Greek models must be imitated to raise the level of German culture. The imitation of Greek models in Art

was destined to be the theme of the first work published by his most famous student.

After three years Winckelmann left Berlin to enter the gymnasium of the Gray Cloister at Salzwedel, from which he entered the University of Halle with the intention of studying theology. The university at this time had about fifteen hundred students and a library of ten thousand volumes and was important in philosophy, theology and law. The Wolfian philosophy was then dominant there as at all German universities. Winckelmann studied philosophy and esthetics under the great Baumgarten, and he also studied Hebrew, mathematics, physics and law. Halle had no professor of Greek, but Schulze, a teacher of medicine and linguistics, admitted Winckelmann to his course on ancient coins. He got but little out of his theological studies except Hebrew; from his legal studies he received valuable lessons on the universality of history, a sense for outlining great epochs and an idea of clearness in exposition, lessons which stood him in good stead in later years. Where Goethe confessed the influence of Kant on his life, Winckelmann's study of philosophy led him to protest against all philosophers except Plato—and Plato was excepted merely because of his redeeming literary style. He never received a degree at Halle nor wrote a dissertation, but contented himself with receiving in February 1740 a certificate of membership in the theological class. For a half year longer he stayed at Halle in charge of the library of the university chancellor, where he spent most of his time reading Greek. He had arrived at the certainty that he was in no wise fitted for a theological career. In after years he spoke disparagingly of his university education, maintaining that he was his own teacher. We would expect such an opinion from a poet, who receives little assistance from a formal education, but from a scholar, such as the historian of ancient art and the votary

of the greatest intellectual tradition, we are surprised at such an admission.

After spending a short time as tutor in a family at Osterburg, Winckelmann in 1741 entered Jena to study medicine. Here he soon found that he had as little aptitude for medicine as he had for theology; his private work of tutoring left him little leisure either for his new studies or for his beloved Greek. The next spring he left Jena and became tutor to the sons of a high Prussian official named Lamprecht living near Magdeburg. Here he met von Hansen, a former secretary of the Danish ambassador to Paris, whose library, rich in modern literatures, was hospitably thrown open to him. From these books the young tutor became acquainted with the French sceptical movement, especially with the *Historical and Critical Dictionary* of Pierre Bayle. After a year and a half Winckelmann received a call as con-rector and teacher of Hebrew, logic and geometry at the gymnasium at Seehausen.

The five years which Winckelmann spent at Seehausen were the dreariest of his life. He always looked upon them in after years as a martyrdom. In one of his later letters we read: "I have enacted the schoolmaster with great fidelity and taught children with scabby heads their a b c's, while during this pastime I was ardently longing to attain to a knowledge of the beautiful and was repeating similes from Homer....At that time I was constantly saying to myself what I still say at the present time:

τέτλαθι δή, κραδίη, καὶ κύντερον ἄλλο ποτ' ἔτλης."[4]

No one in Seehausen could doubt his ability or skill as teacher; but a man whose head was full of such lofty ideas must necessarily have presided over his classes in an indifferent manner. His predecessor Boysen had been a veritable Orbilius and Winckelmann found his pupils de-

[4] From the Odyssey, Bk. 20, l. 18: "Endure, my heart; yea, a baser thing thou once didst bear" (Butcher and Lang).

ficient in taste and far more interested in facts than in sentiment. Boysen, who had become a preacher in Magdeburg, wrote that he could say without self-praise that he "had done incomparably more for literature and the sciences in the year and a half that he had acted as assistant rector than was done in five years by his successor." After his day's work in the schoolroom, Winckelmann had to spend the early evening tutoring Lamprecht's son whom he had brought along with him, and he was only free to do his own reading after ten o'clock. He spent the greater part of the night reading Homer, Sophocles, Herodotus, Xenophon and Plato. He ordinarily retired at midnight but arose at four the next morning to read until six, when his school duties began again. It is said that for a whole year he never undressed to go to bed, but slept in his chair. He literally followed the poet's advice:

"vos exemplaria graeca
nocturna versate manu, versate diurna."

Apart from his Greek authors he also read largely in modern literatures. He even found pleasure in Voltaire's artificial classicism; the subtle Frenchman, whose superficial taste Winckelmann was one day to supplant by the clear ring of the genuine ancient spirit, at least gave him a love for French letters, which contrasted with his contempt for German books. We must remember that Goethe was not yet and that there was nothing in German literature which could have anticipated his *Iphigenie*.

In teaching Greek Winckelmann had little in the way of texts. There were in Germany at this time only a few Italian and Dutch texts of the classics and about the only Greek books for class-room use were the selections of Borst and Gesner. Not satisfied with such ἀποσπάσματα Winckelmann made handwritten copies of commentators and scholiasts. Some of these manuscripts, beautifully written, are still in existence. He planned with a fellow teacher to

publish a collection of classical authors and actually annotated parts of Sophocles as well as Juvenal and Persius, which were never published. He was mercifully saved for something higher than the editing of text-books.

In addition to his poverty—he received only two hundred and fifty thalers a year—overwork and school duties which he hated, Winckelmann also got into trouble at Seehausen with the rector. As assistant rector he was unable to hold chapel himself and so was obliged to listen to the preaching of his superior. Instead of listening to the service Winckelmann would read his Homer in church and was also untactful in expressing his contempt of his colleague's abilities. His remarks naturally reached the ears of the rector, who in retaliation denounced Winckelmann's knowledge of Latin. However there is a letter in existence whose Latinity at least is above reproach, in which years after Winckelmann expressed his contempt of his superior in language which would have done justice to Martial. Among other things he wrote: "I still remember the looks with which I was insulted by a man lighter than the shadow of a cork-tree, and, of all bipeds, the most worthy to be the wiper of Silenus, the most stupid of the gods."[5] In the year 1747 his unhappiness reached a climax: as the poet says,

> "When sorrows come, they come not single spies
> But in battalions."

His school work oppressed him; he had little time for his own studies; his pupils were stupid; the attitude of the rector had become unbearable; he became lonely and melancholy, and his tiny income scarcely met his simple wants; on top of all his mother, to whom he was passionately attached, died. He longed for a change, but did not know

[5] In a letter to Kleinow: "....Haerent infixi pectore vultus, quibus nobis insultavit homo umbra suberis levior, et omnium bipedum dignissimus, qui Sileno, stupidissimo Deorum, a clunibus sit." (Translation of G. Henry Lodge, *Winckelmann's History of Ancient Art*, I, p. 12.)

where to turn. He knew he was unfitted for the Church, for law or medicine; school teaching had become utterly loathsome to him . Only the literature of art pleased him, and he longed to leave Germany and visit the countries of the classical tradition. As he wrote at this time: "It is my misfortune that I was not born to great place, where I might have enjoyed cultivation and the opportunity of following my instinct and forming myself." But he was thirty years old and had not received as yet a single favor of fortune. In 1748 he wrote Count Bünau of Nötheniz near Dresden—the first German historian and the author of a History of the Holy Roman Empire—"for a corner in his library." In his letter he hinted at his unhappy position "in a metaphysical age by which humane literature is trampled under foot," and continued: "Nowadays little value is set on Greek literature, to which I have devoted myself so far as I could when good books are scarce and expensive." Soon afterward we find him ensconced in Bünau's library of over forty thousand volumes, lodged and paid from fifty to eighty thalers a year. He had finally found congenial work.

During the six years which he spent at Nötheniz he made frequent visits to the collection of antiquities at Dresden nearby. Hitherto he had only known the words of Greek poetry; now for the first time he was in the presence of the visible remains of Greek culture. In Dresden he got acquainted with many artists, especially with Oeser, Goethe's frend and teacher, whose culture and knowledge of art were of great assistance to Winckelmann. Through Oeser's influence he finally moved to Dresden where he spent the year 1754-5—the most important and decisive in his life. Here in the Saxon capital he felt at home: for though born in Prussia, Winckelmann was no Prussian: his gentle nature rebelled against the Spartan military discipline and the police system of that despotic land, and

he was fond of boasting that his fatherland was Saxony and that no drop of Prussian blood flowed in his veins.

Dresden at this time was the most cultivated city of Germany. During the reign of the splendor-loving August the Strong (1694-1733) and that of his successor, the art virtuoso August III, the city had become greatly embellished and had reached a prominent place as a cradle of art. August the Strong had made the grand tour and had become captivated by the spirit of the reign of Louis XIV. On his return he had his architect Pöppelmann begin the erection of the Zwinger, the original plan being to make this building the center of a grand architectural display. It recalls the palatial French edifices which had been built as monuments to glorify the reign of the Grand Monarch. The age of Louis had been fond of comparing itself with the Golden Age of Rome; so the Zwinger was intended to embrace the most sumptuous features of Roman palaces, baths and pleasure buildings. The purpose of Rococo art, which we see in part worked out in this building, was to invest even the domestic life of princes with pomp and state, to show to the people the royal cabinet and private office. The Dresden opera and theater were also French; sculpture, however, was here as elsewhere in Europe dominated by the Italian taste of Bernini. The collection of paintings had been founded in 1722, while that of sculpture, mostly formed from the Chigi and Albani collections of Rome, had started with the Brandenburg collection in 1723. These were the only art collections of any importance in Germany. The Sistine Madonna had been brought to Dresden in 1753 just before Winckelmann left for Italy. The art treasures of the city were so rich that the sculptor Cavaceppi, the fellow traveler of Winckelmann on his last journey, could flatteringly say that Dresden might strive for first place with the Capitoline collection at Rome. A colony of foreign artists lived here, as also several native

ones of note. In short, as Winckelmann said, "Dresden is becoming ever more the Athens of artists"—a sentiment echoed some years later by Herder, who called Dresden the German Florence. The seven years which Winckelmann spent in and near Dresden were indeed happy years. These were the years from the end of the War of the Austrian Succession to the outbreak of the Seven Years War in 1756 and were the most peaceful which Europe had seen for a long period. During these years, as Voltaire says in his *Le siècle de Louis XIV*: "Industry bloomed from Petersburg to Cadiz; the fine arts were everywhere in honor; all nations had intercourse with one another; Europe was like a big family which had become united after its troubled days." Nowhere were the fruits of peace better to be enjoyed than in Dresden, which at this time had the most illustrious court in Europe.

Winckelmann was already past his thirty-seventh year and the world as yet had seen no public proof of his ability and learning. He had begun to ask himself with Juvenal:

"Semper ego auditor tantum?"

In 1755, the year that he left Dresden forever, he published the first of the three great works by which he is remembered, his *Thoughts on the Imitation of Greek Works in Painting and Sculpture*. This was followed immediately by a pretended attack and then by a defence of its principles. Winckelmann had studied the simplicity and repose of Raphael's great Madonna and found that the same elements were also characteristic of Greek art. In this first book was the kernel of his fundamental view of art, words which were soon to be memorable in later essays and in his great History of Art: "One must imitate the Greeks and not nature only; for the Greeks knew the secret." This secret was that art should be characterized by "noble simplicity" and "calm grandeur." As sculpture was the chief

product of Greek art, he discussed it most and showed why it was superior to modern sculpture, which was dominated by Bernini's theatrical taste and characterized by strange and uncustomary poses and treatment. The book, though full of obscurities, reached its purpose—the direct appeal from the artificial classicism of the day to the study of the ancients. It was enthusiastically received; every one was amazed at the author's boldness in assailing the prevailing taste. Lessing got from it the inspiration for his *Laocoon*, the book which Macaulay said "filled him with wonder and despair," in which the author analized the boundaries of poetry and the plastic arts, and enunciated the principle that each art was subject to very definite conditions and could only attain its end by limiting itself to its own function. Winckelmann in a few months was recognized as belonging to the first rank of German writers. It was the turning point in his life. Not only do misfortunes come in battalions, but also, even if more seldom, fortune's favor. By this book he achieved not only celebrity, but, best of all, the opportunity to go to Italy. His Dresden sojourn had filled him with an overwhelming desire to see Rome. For here in Dresden it had become clear to him that art was the main interest of his life and that Italy was the only place in which properly to continue its study. His success was all the sweeter because it was unexpected and in such contrast with his earlier years of struggle. From now on we have a different Winckelmann. His *Lehrjahre* are now over: art has become his religion and now that he has attained his freedom and maturity he appears to us, as Goethe said, "consummate, entire, complete in the ancient sense."

As the Saxon court was Catholic the only road to favor at Dresden was through the Roman ecclesiastics. Back in 1751 Archinto, the Papal Nuncio at Dresden, had visited Nötheniz and had suggested to Winckelmann,

who had acted as his guide through the library, that Rome best suited his health and temperament. He had then held out to him the hope of a place in the Papal library and had told him how Cardinal Passionei, an ardent student of Greek, had been pleased with his beautiful Greek handwriting and would be ready to play the Maecenas if only he would accede to the indispensable condition of joining the Roman Church. The bribe was finally accepted and Winckelmann, after a great deal of hesitation, became a Catholic in 1754. Goethe explains this conversion by pleading that Winckelmann was a pagan spirit to whom Christianity was nothing. That Winckelmann had no intention of deception by the disguise is shown by the fact that he had a book by Voltaire in his pocket when searched at the Roman custom house, and that later during his residence in Rome he lived in constant fear of an inquisition. He gives his own version of the affair in a letter to a friend: "It is a love of knowledge, and that alone which can induce me to listen to the proposal made me." In 1760 he had an opportunity of holding a fat office in Vienna if he would only take the tonsure. At that time he answered: "I was born free and I will die free." Doubtless the fact that the Roman Church was in so many ways bound up with pagan grandeur had made this superficial change of heart easy. In any case his religious sentiments were all merged in those of art. As for his embracing Roman Catholicism he would have turned Mohammedan with equal ease if he could have gained thereby a good chance to study antique marbles. On reaching Rome he was mercifully excused from kissing the pontiff's foot, and Benedict XIV assured him of his continued favor. Dresden had proven to be the gateway to Italy. The Elector of Saxony, pleased with his book, promised him a pension of two hundred and fifty thalers, and in September 1755 he started for Italy.

For the next thirteen years of his life Winckelmann devoted himself entirely to the study of art and archeology. It was fortunate that he, like Goethe, had come to Italy in full maturity of mind. The effect of Rome on the poor German scholar was immense. Everything about the Eternal City pleased him—its free artist life, its antiquities, libraries, language, climate and above all the spell of the past. Here there was no bureaucracy, no military, no police. Everything in the congenial atmosphere of the city with its Hellenic affinities made a truly artistic ensemble for him. From a long familiarity with ancient literature his mind had acquired an antique cast; when he reached the Niobe of nations and viewed its ruins and art treasures, he felt as if he belonged not to the present but to the past. He said in the fulness of his rapture: "All is nothing compared with Rome! Formerly I thought I had thoroughly studied everything and behold, when I arrived here, I found I knew nothing." In a letter three years later to a friend in Dresden he says: "In Rome, I believe, is the university for all the world, and I have been purified and tried in it." He also felt that he was in a sense out of place, for he wrote: "I am one of those whom the Greeks called ὀψιμαθεῖς—I have come into the world and into Italy too late." He was pleased with the cordial reception which he received from the Cardinals Passionei and Albani; their democratic ways contrasted strangely with the *hauteur* of Germans of high position, for he was immediately invited to drive and walk with them on terms of the greatest intimacy. His life was one of the utmost simplicity; at first he lived in the artists' quarter; he never went to the theater nor the opera, but went early to bed where he slept undisturbed by the street noises which at this time were worse than in the days of Juvenal. His delicate constitution only allowed him the simplest fare—generally only bread and wine, though he

drank the latter neat like a German. After remaining four years in Rome Winckelmann lodged in the palace of Cardinal Albani, living on his pension from the Saxon prince and another of about $120 from the cardinal. Four years later, in 1763, he was appointed to the high-sounding office of *Commissario della Antichità della camera apostolica*, with oversight over all the antiquities in and near Rome, at a salary of about $180, his pension from Dresden by then having been stopped. In the same year he was also given a clerkship in Hebrew in the Papal library at a salary of $50, a position which entailed practically no work, but confined him from eight to twelve hours a day. Thus his total income in Rome was never over $350, though this amount was enough in those days for a quiet scholar.

At this time Rome was the center of classical studies. The collections of the Louvre, the Glyptothek and the British Museum were not yet in existence. There was little of importance in Berlin or in any German city outside of Dresden. The sculptures of the Uffizi in Florence, mostly from the Roman Villa Medici, were not set up until the end of the century when also the Farnese collection was taken to Naples. In the latter city the finds from the buried towns of Pompeii and Herculaneum had not yet been made public. So Rome was the only place in which to properly study ancient art. It was the day before travels, excavations, reviews and books on art. The only cast collection in existence was the small one gathered in Rome by Raphael Mengs. It is doubtful if Lessing ever saw a copy of the Laocoon when he wrote his famous essay. Bonn University was the first to have a collection of casts, which was made in the early part of the nineteenth century; now not only all the German universities, but many of those in the United States have them. Even in Rome there were no public museums. Only three of the five great Roman collections of the present day existed in Winckelmann's time—those

of the Villas Albani, Borghese and Ludovisi, and these were all under private ownership. The Medici collection was moved to Florence a little later; the present Capitoline collection was originally in the Villa Albani and after its sale the present Villa Albani collection was begun; the present Vatican museum of antiquities, now the finest in the world, had as its beginning in Winckelmann's day the statues of the Belvedere collection, which had been begun in the sixteenth century by Pope Julius II and was named from the garden house in the Vatican grounds where they were exposed down to the beginning of the eighteenth century. Only a few, however, of the present masterpieces, like the Hercules Torso, the Apollo and the Laocoon, date their appearance in the Vatican from that period. The Museo Pio-Clementino was started at the end of the eighteenth century, while the Chiaramonti and the Braccio Nuovo had their beginnings in the nineteenth.

It was in the year 1758 that Winckelmann made his first visit to Naples to visit the sculptures there and to view the recently opened excavations in the neighborhood. At Resina, on the site of Herculaneum, the theater had already been laid bare and at Pompeii a portion of the amphitheater and the eastern end of the town had been excavated. He stayed in and around Naples for two months, enjoying everything he saw and did, even the eating of enormous cauliflowers and the drinking of Lacrima Christi. He also went on to the site of Paestum, which at that time was merely a malaria-stricken wilderness containing a few shepherds' huts. Here he saw the first Greek temples. Their existence until a short time before had been a secret even to artists and antiquaries. Macaulay, in describing Addison's visit to these ruins at the end of the seventeenth century, graphically writes: "Though situated within a few hours journey of a great capital where Salvator had not long before painted and where Vico was then lecturing,

these noble ruins were as little known to Europe as the ruined cities overgrown by the forests of Yucatan." Winckelmann made in all three more visits to Naples in the years 1762, 1764 and 1767. It was in 1760 that the statue of Diana had been discovered inside a little temple at Pompeii, the first example of an ancient sulpture which retained traces of color. As the fruit of his second and third visits Winckelmann gave to the world in two letters the first authentic information about the excavations at Pompeii and Herculaneum.[6] During his last visit, he, like the elder Pliny, was able to witness a great eruption of Vesuvius. Accompanied by von Riedesel, he went to Portici, whence the party walked out over the ancient lava beds to the new and was compelled, in order to reach the crater's mouth, to pass over hot lava which scorched the soles of their shoes. He had also planned during his first journey south to make a tour of Calabria and Sicily. A journey to Southern Italy, however, was no easy task. The conditions of travel were barbarous; the roads were nearly impassable and were beset by theives and cutthroats. In the Kingdom of Naples one could only go on foot or on horseback and had to be accompanied from place to place by a soldier. If one had no servants and no letters of introduction to landed proprietors along the way, he had to put up with the food of an anchorite and to sleep on pallets no strangers to vermin. In a letter Winckelmann recounts how his journey to Paestum in 1758 was filled with a hundred annoyances. Of the danger of brigands he says: "One must go with two pistols in his sack, two in his girdle, and with a good claymore at his side and a gun on his shoulder." Despite the ridiculous figure the poor scholar must have cut in such a panoply, he says he bought all these necessaries in Naples. The ignorance

[6] *Sendschreiben von den herculanischen Entdeckungen* (1762) and *Nachricht von den neuesten herculanischen Entdeckungen* (1764).

which prevailed among educated men of that day about Calabria is shown by Winckelmann's belief that there were ruined temples there. He only gave up his intention of visiting them when he learned from the English noble Brudnell, who had just returned from a journey along the coast as far as Taranto, that, outside the temple of Juno at Croton, there were no ruins to be seen. In his last journey to Naples he also again seriously had in mind a trip to Sicily to visit the Doric architectural ruins there. His enthusiasm had been fired by the descriptive letters written by Riedesel, who was the first scholar to make the island known to lovers of art. Goethe, years later when in Girgenti (1787), spoke of Riedesel's little volume, which he says he carried about with him "in his bosom like a breviary or talisman." At the beginning of 1760 Winckelmann also seriously considered a trip to Greece with Lady Oxford. He then wrote: "Nothing in the world have I so ardently desired as this; willingly would I allow one of my fingers to be cut off, indeed I would make myself a priest of Cybele, could I see that land." Again in 1768, the year of his death, he was invited to accompany von Riedesel to Greece. But he was destined never to see either Sicily or Greece. New vistas of travels and plans for work were constantly being opened up to his mind; the infinity of possibilities made him sadly reflect—

"Ach, das Leben ist am Ziele
Und die Kunst noch kaum begonnen."[7]

During his first visit to Naples in 1758 Winckelmann had been recalled to Rome by the last illness of the Pope and immediately after his death went to Florence, which he described as "the most beautiful place I have ever seen and far superior to Naples," and as "the true cradle of the Italian art spirit." While here he studied the art treasures of the city and worked assiduously on a catalogue of the

[7] "Alas that life has reached its goal
And art is scarce begun."

great collection of gems owned by the Prussian Baron von Stosch, who resided there. He says that he never before had worked so hard; for six months he only allowed himself a half-hour's relaxation in the evening. He had to complete the catalogue in Rome the following year, where he could avail himself of the study of the gems in the Museo Kircheriano and of the advice of connoiseurs in that field. The work finally appeared in 1760 under the title *Descriptions des pièrres gravées du feu Baron de Stosch*—his first scientific work. It was while he was in Florence that his old friend Archinto, the secretary of Cardinal Albani, died, and he was summoned to Rome to become the librarian and companion of the aged prelate. It was after this that he wrote many essays on various phases of the subject of art and antiquities; he also carefully studied the descriptions of monuments in Pausanias and the conception of the *Beautiful* in Plato. Many of these minor writings—like those on the *Apollo Belvedere* and on *Grace in Works of Art* and the study of *The Capability of the Beautiful in Sculpture*—are among the most beautiful from his pen. But the results of all his studies and writings finally culminated in his greatest work *Die Geschichte der Kunst des Altertums*. It will be convenient at this point in recounting the chief events in Winckelmann's career to briefly bring together what relates to the origin and fate of this work and also of his last book the *Monumenti antichi inediti*.

Winckelmann had had the plan of writing the *History of Ancient Art* in mind ever since his second year in Rome. He continually visited the treasures of the Belvedere to arouse his spirit, and from these visits grew his desire to write such a book. He looked upon all his preceding reading and essays as merely preparatory to this work, which for years robbed him of most of his time. He was long in doubt in what language such a history should be written. Cardinal Albani suggested Italian on the theory

that *dum vivis Romae, Romano vivito more.* But Winckelmann finally decided on his mother tongue. The first draft of the work had been sent to Dresden for publication in 1758, but, in consequence of delay, it had been withdrawn the following year. This proved to be a fortunate circumstance, for it allowed the author to recast it and to produce an almost entirely new work. This revision extended over the years 1758-61, and in 1762 he again looked about for a printer. The work finally appeared in Dresden in two volumes quarto in the year 1764 and was dedicated to Friedrich Christian, the Elector of Saxony, who had succeeded August III, Winckelmann's patron, the year before. The size of the edition made it impossible to bring out a new edition for some time, so that the author had to content himself with collecting emendations and additions for a second work entitled *Anmerkungen über die Geschichte der Kunst,* which appeared in 1767. Just before his death he had begun to recast the material for a second edition which was to appear in French; but fate was against him. On his last journey he carried the manuscript for this edition with him, and the very last words that he penned while in Trieste, where he was murdered, were in reference to it. After his death the manuscript was sent to Vienna, where it was published with great negligence. From this publication came the Italian edition and another French translation.

Winckelmann's *History of Ancient Art* is the earliest work in which the origin and development of sculpture and painting in Egypt, Phœnicia, Persia, Etruria, Greece and Rome is systematically presented in connection with the general progress of culture. Following the custom of French writers on art, he wrote an art history in general, but one of Greece in particular. He recognized that art was but one phase of the history of mankind, though it was the flower of national life and evolution. Not con-

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tent with merely presenting the beautiful monuments of art, he investigated the sources of beauty in Greece and the reason why Greek art still commands the world's admiration. In unfolding the theory of the *Beautiful* he finds that "the highest purpose and the central point of art" is beauty rather than instruction. This thought was to dominate artists, critics and poets for the next two generations. Ideal beauty can only be attained when individual features are subordinated to the general scheme in the mind of the artist. The artist selects his theme from the natural world and combines it with his imagination, thereby creating an ideal type marked by the two characteristics of "noble simplicity" and "calm greatness" or "repose." All details, like muscles and veins, must not be allowed to impair the harmony and the proportions. He discerned for the first time what is now a commonplace of knowledge that "beauty has been esteemed by no people so highly as by the Greeks." The beauty of the youth of life was so extolled by the Greeks that Aristobulus in Xenophon's *Symposium* is made to say: "I swear by all the gods that I would not choose the power of the Persian king in preference to beauty." This Greek ideal of beauty is nowhere so preeminent as in sculpture, where it is especially associated with youth, for in youth more than in manhood the artist finds the causes of beauty," in unity, variety and harmony"; "the forms of beautiful youth resemble the unity of the surface of the sea, which at a distance appears smooth and still like a mirror, although it is constantly in movement with its heaving swell."

In writing his History Winckelmann used everything —monuments and books both ancient and modern. His own artistic sense, helped on by vast erudition and by a vigorous imagination, enabled him to make remarkably true suggestions about periods of Greek art where little real information then existed. He overthrew many of the

older interpretations of monuments, which had been based on the false theory of the Roman origin of ancient art. Thus he found that the portrait busts in Italian collections were far too realistic to be Greek, too much out of harmony with Greek ideality. He was the first to divide Greek art into epochs, indicating the sequence of styles corresponding to changes in society and politics. These divisions are still kept in our histories of art; the archaic style (*älterer Stil*); the grand style (*hoher Stil*) of the age of Phidias, characterized by grandeur, beauty and truth to nature; the beautiful style (*schöner Stil*), beginning with Praxiteles and characterized by elegance and grace; and lastly the style of the imitators, when the old ideals of simplicity were lost and a pretentious and decadent taste came in. At the end of the work he devotes a few pages to Roman art, a period in which all originality had been lost and art was devoted to the repetition of earlier types.

There was really no one in 1764 who was able to criticise adequately this work. The few who knew Greek literature knew nothing of the monuments and those in Rome who were acquainted with the latter knew little of Greek letters or history. The work was nothing short of a revelation to his contemporaries and it profoundly influenced the best minds everywhere. It was praised by learned societies and scholars for its flowing style, its erudition, its sane judgments, its insight and its sense of beauty and proportion. It was soon recognized as a permanent contribution to European science and *belles lettres*. Lessing received a copy while still at work on his *Laocoon*, and was unbounded in his praise; the contemporary French sculptor Falconet said he had "read nothing better on the beautiful in art"; Diderot was more guarded, for while praising the author's enthusiasm, he felt the application of his ideas to sculpture was wrong, since he did not agree with Winckelmann's fundamental notion of art, that it

should imitate the antique rather than nature. The Italian architect Visconti and the Frenchman Quatremère de Quincy years later found nothing but praise for it; Madame de Staël, in her *Allemagne,* said that it was Winckelmann who "brought about an entire revolution in the manner of considering the arts" and that he had "banished from the fine arts of Europe the mixture of ancient and modern taste" and that "no one before him had united such exact and profound observation with admiration so animated."[8] Winckelmann's Roman contemporaries, like Raphael Mengs, were unbounded in their approval. Heyne in Göttingen some years later wrote a eulogy of the author, though he, like Diderot, tempered his praise with real criticism. Heyne had written on Pliny's art epochs and was surprised that Winckelmann had made so little use of that author; but the author of the *History of Ancient Art* knew that Pliny was no evangelist in matters of art. Heyne called attention to the weakness of the work—its uncritical statements and inaccuracies, though he was wrong to conclude that the historical part was therefore "practically useless."

The *History of Ancient Art* was a masterpiece of German prose; though primarily a scientific work, it possessed all the grace, rhythm and dignity which we expect in a work of pure literature. With Lessing's Essay it may be said to be the beginning of modern German prose. These two writers brought German literature into line with the world literatures and by opening to the Germans the empire of beauty brought a plastic element into their poetry. Winckelmann confessed that he had followed the dictum of Roscommon that the "greatest masterpiece of everything in which mankind has been distinguished is good writing." The style, always original, is at times grand as when treating of the essence of beauty and in certain descriptions

[8] See her eulogy, *Allemagne,* Part II, Chap. VI (transl. by O. W. Wright).

it actually soars. Of his eloquence in describing the Apollo Belvedere and the Laocoon, Madame de Staël found his style as "calm and majestic as the object of his consideration." But its style and poetic beauty are its least important features. It instituted the historic study of art and indicated the methods by which that subject must be approached; greatest of all, it overthrew the false taste of the day and for the first time scientifically showed the existence of an independent Greek art.

In criticising the contents of this work to-day we must bear in mind that it entered an almost new field of criticism and therefore was influenced but little by anything which had preceded it; furthermore we must remember that it was composed at a time when but few monuments of the great period of Greek art were known. In the preface Winckelmann mentions the now forgotten works of the painter Monier, of Durand and of Turnbull. The best preceding work on ancient painting, that of Franz Junius, which had appeared well over a century before (in 1637) and remained the source for the study of Greek art all that time, he does not mention. This work was, however, more philological and philosophical than historical in character and had been written by a man who had lived most of his life in England and who had never seen Italy. The Frenchman Goguet published in 1758 a work on the *Origin of the Laws of the Arts and Sciences*, "one of the best books of our times," as Winckelmann termed it; but this work was anthropological and historical in character rather then esthetic. The most exact and learned work on Greek sculpture was a part of the recent *Recueil d'antiquités* of the Compte de Caylus,[9] who had traveled extensively in Italy, Greece and the East. As for the monuments of Greek art then available to Winckelmann in Italy,

[9] *Recueil d'antiquités égyptiennes, étrusques, grecques, romaines et gauloises*, 6 vols. Paris, 1752-5.

it may be said that the most important examples now known to us had not yet been discovered. Of the archaic period there were but few significant works and almost none of the time of Phidias and little of the fourth century B. C. Still Winckelmann's treatment of the "grand" and "beautiful" styles of the fifth and fourth centuries are for all times and all peoples.

Many of his historical conclusions about art are, of course, mistaken. Thus his idea that the Greeks first worked in clay and then in wood, ivory, stone and bronze successively must be given up, as well as his idea that the Greek sculptor first used unwrought cornered blocks, which were subsequently rounded and then fashioned into herms by placing heads at one end and later differentiated by sex, followed by sculpturing the upper part of the body and then the lower, until at last Daedalian statues with the legs separated were evolved. He was quite as mistaken in his contention that Greek art was independent in origin, quite uninfluenced by the art of Egypt and the East, the Greeks "not deriving the first seeds from elsewhere," but "appearing to have been original discoverers." We must also remember that Winckelmann had to reach Greek art largely through Roman copies and imitations; consequently many of his conclusions are inadequate in their basis and have been either completely overthrown or largely modified by subsequent discoveries. Thus no one to-day would echo his excessive praise of such monuments of sculpture as the Laocoon, the Hercules Torso or the Apollo of the Belvedere. He assigns the Laocoon to Alexander's time, but concludes that posterity has been "unable to produce anything worthy of being compared with it even remotely." The torso he places in the age of Alexander's immediate successors and looks upon it as the "lofty ideal of a body elevated above nature"; the Apollo belongs to the imperial times after Nero's reign and is "the highest ideal of art

among all the works of antiquity which have escaped destruction," and the effect produced on him by its aspect is "indescribable." He even explains the lack of veins in both the Torso and the Apollo as a sign of their "heavenly essence." The excellencies which he saw in these and similar works we can now see in far less contaminated purity in many monuments which were unknown in his day, and consequently we judge them from a very different standard when we compare them with genuine products of Greek art of the great period instead of with those of the decadent epoch. If Winckelmann had seen such beautiful statues at those of the Hermes or the Melian Aphrodite, the pride of the Louvre and by many looked upon as the most beautiful of all sculptures, their "noble simplicity" and "calm greatness" would have called forth the encomium which, in the absence of such noble works, he gave to decadent pieces. In that case he doubtless would have seen in the Belvedere torso not a resting Hercules at all, but perhaps merely a Cyclops—as Sauer maintains— who is holding up his hand to shade his eyes as he looks out over the sea to get a glance of his beloved Galatea, and so this piece, with all its fine modelling, would fall into place among Pergamene works of the Hellenistic period; its lack of veins, then, could not be explained by an attempt to deify a hero or to etherealize his body. Lessing, instead of using the Laocoon as an example of what sculpture should not attempt—for it not only groans as he and Winckelmann said, but shrieks—uses it to illustrate the difference between the principles of poetry and sculpture. If either had seen the masterpieces of sculpture from the Elgin marbles down, they would have judged it very differently and seen that it, like the Laocoon, belongs to Pergamene art, as an extreme example of the tendencies of that art toward dramatic power and exaggerated pathos. In the case of the Apollo, however, we must admire Winckelmann's insight; for

many modern critics believe it is a copy of an original bronze dating, perhaps, from the fourth century B. C.; its original has even been assigned to Leochares, who worked with the great Scopas on the Mausoleum.

To one so imbued with the Greek essence of beauty, it is not strange that Winckelmann denounced the fantastic and exaggerated conceits and affectations of modern art and in opposition fearlessly preached his admiration of the purity, naturalness and simplicity of ancient works. But it was just this insistence on Greek ideals that at times led him into wrong appraisals of certain modern artists. Thus his wholesale condemnation of the greatest of the Renaissance sculptors seems to us not only harsh but wholly unjust, even if we try to excuse it by the fact that he doesn't judge him from the point of view of modern artists but from that of the ancients. In a classic simile he says Michelangelo compared with Raphael is what Thucydides is compared with Xenophon. To him the supreme interpreter of the Old Testament, the immortal artists of Sibyls and Prophets, by his striving after the difficult and extraordinary, and his "studious employment of scientific knowledge," is merely the originator and promoter of the corruption of taste, which culminated in the theatrical motives and strained attitudes of Bernini's art. While admitting that he "contemplated lofty beauty," he finds this feature in his poetry rather than in his sculpture and painting. His Christ heads are "mean and vulgar" and "borrowed from the barbarous works of the Middle Ages." The youthful beardless heads of Christ painted by Raphael and Annibale Caracci, as well as the bearded Christ of Leonardo, he found far more noble. Winckelmann's insistence on Greek ideals led him to affirm that subjects drawn from the Christian religion were not favorable to art, and consequently he endeavored to arouse in the artist enthusiasm for classical mythology. Thus he said that artists should

copy their Saviours from the models of Greek heroes and their Holy Virgins from Amazon heads, not perceiving that any such slavish tendency would mean the deathknell to all independence and progress. In violent contrast to his disapproval of Michelangelo, he lauds his friend Raphael Mengs as "the most accomplished instructor in his art," and speaks of the "immortal works" of him who had "reached the highest point of excellence to which the genius of men has ever risen." He ends his panegyric of Mengs by calling him "the German Raphael." We are reminded that a German admirer of the author of Paradise Lost called Klopstock "the German Milton," and that Coleridge sneeringly rejoined "a very German Milton, indeed!"

Winckelmann's second great work was written in Italian, the splendidly illustrated *Monumenti.* On his forty-fourth birthday in 1761 he announced this work which finally appeared six years later. This "classic work," as Visconti called it, was chiefly intended for Italian scholars and lovers of art and not, like the *History of Ancient Art,* for the general reader. It was more the fruit of Winckelmann's Italian sojourn than any of his other works, a fitting tribute of the author to his adopted land. Casanova furnished the drawings for the more than two hundred copper plates and vignettes, which were mainly taken from sarcophagi reliefs; the expense of draftsmen, engravers and printing were all borne by the author. The plates, selected from unpublished monuments, were accompanied by explanations of mythology, customs and history. Winckelmann spent much time and energy on this monumental work. He says in a letter: "It is known to God and myself how I have sweated over it. There are pieces in it over each one of which I have sat for five months." In recent years it has been objected that the work was overloaded with unnecessary learning after the Italian fashion, on the assumption that the author wished to make

a display of his erudition among his Italian contemporaries. In any case it is an invaluable work and shows the same original and independent style which we see in all his writings.

We now come to the last scene in Winckelmann's life—his untimely end. On leaving Germany for Italy thirteen years before, he had had no intention of remaining there permanently. But soon after he left Dresden the Seven Years War broke out and Saxony, especially the capital city, suffered terribly. He was fortunate to receive his pension at first; but after two years it was cut in two, and in 1763, on the death of his patron August II, it was withdrawn entirely. In the winter of 1767-8 on returning from his last visit to Naples, he was hard at work on the revision of his History, for he intended in the spring, in conformity with a plan which he had long had in mind, to revisit Germany and especially Berlin, where he was to see a French edition of his work through the press. The recent invitation extended to him by Frederick the Great, to come there and take charge of the royal collection of antiquities, was well known in Italy, and every attempt was made to dissuade him from going, as it was felt that he was the only man in Italy with a critical knowledge of Greek literature and art, and it was feared that he might never return if he again visited his native land. It was also just at this time that von Riedesel invited him to accompany him to Greece and the East, a journey which Winckelmann had longed for all his life. It was a far easier thing for him to get permission to go to Greece than to cross the Alps. It was hard for him not only to refuse Riedesel but to break the ties which bound him to his Roman friends, especially to the aged Cardinal Albani. Still the desire to see his old home finally decided him to go north. In his last letter to his old friend Franke at Nötheniz he fondly referred to the *Ruheort* where they were to

meet; but he was destined to see neither him nor any of his other early friends.

He started north on April 10, accompanied by his friend the sculptor Cavaceppi, who has left us a description of a part of the journey. They traveled via Bologna, Venice and Verona, and all went well until they reached the Tyrolean Alps. It was here on his journey to Rome years before that the grandeur of the mountain scenery had delighted him so much that later he regarded this part of the journey south as the most agreeable; at that time he had written to his friend Berends: "I should fill my whole letter with things about the Tyrol, if I should attempt to describe the rapture into which I was thrown." Now all was changed; thirteen years in Italy lay between. He now looked upon the same nature with aversion, calling it a "shocking, horrible landscape," and he even found fault with the architecture of the picturesque thatched Alpine chalets. He told Cavaceppi he could not find words to express his feelings of aversion. His companion at first thought he was jesting. In a few days they reached Munich and finally Ratisbon, where Winckelmann came unalterably to the determination not to continue the journey, but to return at once to Italy. Though he recognized that Cavaceppi's remonstrance was just and that he was leaving him in a country whose customs and language he did not understand, he answered that he felt "an overpowering impulse within him which he could not withstand," and immediately wrote Cardinal Albani his intention of returning. Only with the greatest difficulty was he prevailed upon to return by the longer route via Vienna. On reaching the Austrian capital Winckelmann was received with great honors, and Prince Kaunitz tried to persuade him to renounce his determination. His emotion grew so great that he lay sick of a fever for days and finally Cavaceppi gave up hope of dissuading him and left. On his recovery he had an

audience with Maria Theresa at Schönbrunn and received from her and Kaunitz several gold and silver medallions as tokens of their regard. A promise was even exacted of him to return the next year to arrange the empress's cabinet. But Winckelmann was counting the days before he could go; he wrote the young Baron von Stosch in Florence that there was no pleasure left for him in this world outside Rome.

To many this sudden determination of Winckelmann to abruptly terminate his long-planned journey has seemed inexplicable. The circumstances of the last few months of his life explain it only in part. He was certainly worn out with his arduous work; two years before he had suffered from fainting fits, and had gone to Anzio for rest, and in March of the present year he had had a recurrence of the same malady, which, as he said, warned him "to bring his house into order." He suffered also from weak eyes and stomach. The fatigue of weeks of post-traveling through scenery which he no longer cared for aggravated the annoyance caused by suddenly breaking into the quiet of his Roman life. The contrast between the joyous Italian primavera and the bleak and lonely Tyrolean and Bavarian mountains brought on a Roman homesickness. Doubtless the memory of the hardships of his youth also came back to him as he approached his old home. But all these things together do not explain his feelings, for they could not have affected to such a degree a strong and healthy nature in the prime of life. However, it is not necessary to see anything mysterious in his decision, a kind of presentiment of evil which came to him in his weakened nervous and physical condition, even if many sentences in the letters of his last few years speak of his expected early death. This Italian homesickness is by no means an uncommon phenomenon. While to most of us the yearnings which draw us to the ancient world remain faint and remote, to

Winckelmann they were strong and insistent. As Madame de Staël says: "He felt himself attracted with ardor toward the South; we still find in German imagination some traces of that love of the sun, that weariness of the North (*cette fatigue du nord*), which formerly drew so many northern nations into the countries of the South. A fine sky awakens sentiments similar to the love we bear our country."[10] Zimmern, in his recent book on *The Greek Commonwealth*, has expressed a similar thought when he says that one must enter deeply into the spirit and life of the south before one realizes the difference in outlook. Even northern poets who have sung of the Southland have done so for the most part as visitors to whom the real spirit of the country has remained largely exotic, even if it arouses their enthusiasm. The gulf in most cases is not bridged by a lifetime: often a northern invader of Greece would finally return home because of homesickness. Many a Frankish baron of medieval Greece left his domain to go home and die by the Loire or Rhine. Thus Otto de la Roche, the first feudal lord of Attica, who "had the Acropolis for his castle and the Parthenon for his minster" left all in his old age and with his son returned to Burgundy to die. Just so Winckelmann swayed between the desire to see the land of his birth and to return to the land of his adoption. His real home was Italy and not the flat steppes of Germany; he was, to quote the words of Goethe, "of an ancient nature reappearing, so far as that is possible, among his contemporaries."

He reached the port of Trieste on June 1, whence he intended to take ship for Venice. The closing scene[11] of his life drama took place in the *Grosser Gasthof* on the Petersplatz. In the next room to his was lodging an Italian

[10] *Allemagne, loc. cit.*, (Wight's translation).

[11] See the little book by von Rosetti entitled *Johann Winckelmann's letzte Lebenswoche* (Dresden, 1815).

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adventurer named Arcangeli, who was formerly a cook and who four years before had been sent to prison as a thief. This man had come to Trieste on foot and without luggage and was also awaiting a chance to return to Italy. The two men became companions at table and the Italian volunteered to aid Winckelmann in finding a ship. During the week of waiting the two were constantly thrown together at table, and Winckelmann asked the Italian to visit him in his room, and they also took walks together. It seems strange that such an intimacy could have grown up between scholar and peasant; but Winckelmann wanted to remain incognito and was glad to while away the tedium of the days that passed in talking his beloved Italian, and Arcangeli pressed the acquaintance for his own purpose. With characteristic frankness Winckelmann had shown him the medallions which he had brought from Vienna. The avarice of the Italian was at last aroused by these paltry souvenirs. The last morning while Winckelmann, without coat, cravat or wig, was seated at his table writing a letter, Arcangeli entered his room and the two spent a half hour walking up and down conversing. Winckelmann invited his companion to visit him in Rome and promised he would then disclose to him his identity and show him the palace in which he lived. His mysterious hints as to who he was aroused the suspicions of the Italian, who concluded that he was either a Jew or a Lutheran or perhaps a spy. After returning to his own room, he put a knife into his pocket and again entered Winckelmann's chamber on the plea of recovering his handkerchief. He then asked him again if he would show the medallions at the dinner table and, on Winckelmann's refusing once more, asked him why he was so reticent about his identity. Winckelmann, offended at his impertinence, did not answer, but reseated himself and began to write. Then Arcangeli quickly threw a noose over his head, dragged

him to the ground and stabbed him five times in the chest and stomach. A servant, aroused by the uproar, rushed in and found the Italian over the prostrate body of Winckelmann, who was groaning deeply. The murderer forthwith ran hatless out into the street. Winckelmann lived for six hours, during which he dictated his will and received the last offices of the Church. In his traveling chest were found his favorite authors—Homer, Plautus and Martial. He was buried in the plot of a brotherhood in the churchyard of the cathedral of San Giusti. Later, when his remains were crowded by new arrivals, his bones were cast into the common charnel house. It is pleasing to know that the cowardly assassin was soon caught on the Italian frontier and was brought back to Trieste and tried, and six weeks later, on the same day and at the same hour in which he committed the murder and before the window of the hotel where it had occurred, suffered the punishment of Ixion.

Thus Winckelmann departed from life as poor as he had entered it. But behind him lay his brief, though glorious, life of struggle and service. A more fearful end can scarcely be imagined. The gods, however, were kind to him, for they brought him death near the border of the two countries to which he, half German, half Italian belonged. He was only fifty-one years old and therefore still in the prime of vigor. In the beautiful words of Goethe "he had the advantage of figuring in the memory of posterity as one eternally able and strong; for the image in which one leaves the world is that in which one moves among the shadows." Goethe, then a lad of nineteen, just leaving the University of Leipsic for Strasburg, was eagerly awaiting the promised opportunity of meeting the great Hellenist, when he received the tidings of his death. In a letter which he wrote years after in Rome (1786), in speaking of the emotion which he felt on reading some of the cor-

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respondence of Winckelmann which had come into his possession, he said: "How bravely and diligently did he not work his way through all difficulties; and what good does it not do me—the remembrance of such a man in such a place." Walter Pater calls it a calamity that the expected meeting of these two never took place, for thereby German literary history lost a famous friendship. Though a bust of Winckelmann was set up in the Roman Pantheon only four years after his death, no monument marked the place of his passing until fifty years had gone by, when a beautiful statue was erected in the square of Trieste. It was almost a century before his native Stendal set up a monument to its greatest citizen. In 1805 Goethe wrote his *Winckelmann und sein Jahrhundert,* the title of which rightly appraises the European position of this famous scholar; in 1865-72, a full century after his death, Karl Justi gave to the world the first accurate account of Winckelmann's short life. In these latter years he has received the full meed of honor which his abilities and influence have merited.

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CRITICISMS AND DISCUSSIONS.

THE MECHANICAL EXPLANATION OF RELIGION.

There has been a common opinion in the past that what, in a broad sense, is known in philosophy as mechanical explanation—that is, explanation by antecedent cause,—is absolutely opposed to teleological explanation. According to this view, if some theory of a mechanical form is true, any teleological theory (concerning the same explicandum) must be false, and vice versa. This opinion is even still not uncommon, notwithstanding the philosophy of Leibniz and Kant's third critique. To me the contrary view appears to be correct, and the recognition of its truth to be very important, especially in the treatment of religious phenomena. A detailed argument in support of this contrary view has already appeared in this journal from the pen of its editor, Dr. Carus.[1] In the present article I propose to offer an analysis of the situation, from a somewhat different standpoint, in further support of this theory.

Since the time of Fichte, the dominant school of speculative thought has tended toward explanation that is teleological. With regard to mechanical explanation two attitudes have been adopted. It has been said, on the one hand, that mechanical explanation, accurately carried out, is true "so far as it goes," but that it is not the whole truth, and that in particular, it must be supplemented by teleological explanation. On the other hand, the position that mechanical explanation is *no* explanation seems to have been held not infrequently. Advocates of this view would probably admit that to many physical phenomena no teleological explanation can reasonably be assigned; and, in so far as they held that explanation

[1] Vol. XXIII, No. 2.

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must be either mechanical or teleological, they would consequently have to admit that of such physical phenomena a mechanical explanation is the alternative to no explanation at all. They would be disinclined, nevertheless, to accept such explanation. And they would resent any attempt to explain mechanically something with regard to which they believed themselves possessed of a teleological explanation. This happens most frequently concerning things that are judged to be valuable, and perhaps most conspicuously in connection with religion.

Even those who admit that mechanical explanation is true "so far as it goes" often appear to share this resentment toward attempts to treat religion from the mechanical point of view. This is not because such attempts have always issued in explanations which were *mechanically* inadequate, even if this be true. Any thinker, even though he were to believe in the universality of mechanical explanation, would object to such explanation in so far as it was inaccurate. He might resent any general acceptance of the proposition that water under normal pressure boils at 211° F. And he might be willing to admit that the mechanical explanation of religion which sees its origin in the lust of priests and the tyranny of kings cannot be accepted as remotely probable. This, however, is not the attitude of the teleologist. He says, in effect, that such a phenomenon as religion, being of vital importance to man, must be explained by its function, not by its cause; and that its significance is destroyed by any theory which is in essence mechanical. He objects to "the Enlightenment" treatment of religion, not because it was inaccurate in detail, but because it was wrong in its *form*. And his position seems at first sight to be borne out by modern philosophical speculation.

What is usually regarded as the idealistic attitude toward mechanical explanation, when this is offered as ultimate philosophical hypothesis, seems now established. We obviously cannot explain the whole of existence by something, as it were, antecedent to it. The alternative is for mechanical theory to explain *part* of the whole by an antecedent and "ultimate" *part*. The objection which then finally emerges is that the explanatory reality,—whatever form it assumes,—remains *mechanically* inexplicable. Explanation has been obtained perhaps even at the cost of creating a new explicandum.

It seems to be sometimes assumed that the only conclusion to be drawn from this position is that philosophical explanation must

be teleological; but the considerations which make mechanical explanation finally impossible, make teleology impossible also. We certainly cannot explain the whole of existence by an end that is its consequent,—which is somehow to come after it, and is, at any rate, additional to it. There is nothing after, just as there is nothing antecedent to, the whole of existence. The alternative here also is to explain (teleologically) one *part* of the whole by another *part*. But this is formally no better than the analogous situation with regard to mechanism. At least, this is so if "explaining a thing teleologically" be defined as "showing that it is a means to the realization of something else." For the teleologist's "end" is a final inexplicable just as is "the first cause" of the mechanist.

It of course might be held that we give a teleological explanation of anything if we show that it is *either* means or *end*; and upon such a view we have, in showing that one thing is means to another, given a teleological explanation of both things. But to such a theory the advocate of mechanical explanation could reply that he, in his turn, will hold that both the explicandum and its cause are (mechanically) explained once the latter has been assigned. The teleologist could not reasonably deny the justification for this procedure without renouncing his own formally similar procedure. It must therefore be concluded that if ultimate philosophical explanation cannot be mechanical because it involves the postulating of a reality not mechanically explicable, an analogous reason leads to the abandonment of the idea that such explanation can be teleological.

The teleologist may endeavor to escape from this position by suggesting that everything is to be explained as both means and end, the significance of this suggestion being that everything is teleologically explained in so far as everything is a means. Apart from the difficulty of proving such a theory, the formal difficulty still remains. No doubt everything is explained so far as everything is means; but, on the other hand, nothing is explained so far as everything is end. The explanation still leaves an inexplicable, an inexplicable, indeed, which now permeates the whole.

The result which thus appears to be forced upon us by considering the formal characteristics of the two methods of explanation, is corroborated by an impartial survey of actual theories embodying them. If it must be admitted, in the face of the teleologist's charge, that the mechanical "first cause" alternates between

possessing more content than can be justified and possessing no content at all; even if it must be admitted that this explanatory reality always tends to approximate to the empty abstraction, the thing-in-itself; it must be charged to the account of the teleologist that his professedly complete explanations are purely verbal. The theory that all the parts of the universe are means to a certain end, such as the realization of self-consciousness or spirit, is plausible, if at all, only so long as we keep to the most general form of statement. Descend to particular facts; inquire what evidence there is for believing that the realization of self-consciousness or spirit demands, for example, just the existing number of human beings, or of birds, or of trees; and the whole "explanation" is seen to be "on paper" only.

It must therefore be concluded that when regarded as ultimate philosophical explanation, theories of mechanism and theories of teleology are, formally and materially, equally unsatisfactory. The recognition of this fact has been stated in the form that of the universe as a whole no explanation can be given. According to this view, which appears correct, the totality of existence is the one great inexplicable which must simply be accepted; and the most philosophy can do is to illuminate its nature.

So far as philosophy is concerned, the interesting question at this point is what form this illumination of the nature of the universe is to take,—in other words, precisely what is the problem of philosophy. The importance of this question is due to the fact that no one can be expected to solve a problem which is not definitely stated. It is outside the scope of the present article to deal with this point further than to emphasize its importance. For it is in fact partly due to the neglect of it that the conflict between the teleologist and the mechanist breaks out once more, but in a form somewhat different from that already considered. Whether or not the attempt to give an explanation of "the whole" is now definitely abandoned, the actual problems dealt with are of a more modest character. Each side clings to its form of explanation as the vitally important one, and, applying it to one finite phenomenon after another, endeavors to extend its range indefinitely. Attempts are made to show, on the one hand, that the category of means, on the other hand, that the category of antecedent cause, is of universal application, each position being stated as in some sense a methodological principle, while the presumption in favor of each is con-

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sidered to vary directly with the number of phenomena explicable, and inversely with the number not explicable, by it.

When the two positions are thus opposed, it may seem difficult to decide between them, or to see on what principle any decision is to be reached. In practice the universalization of each type of explanation is fraught with difficulties, upon which its opponents fasten. Mechanical explanation appears to proceed smoothly so long as it keeps to the inorganic sphere. Immediately it leaves this sphere, its task becomes harder, and the admission must be made that of much that is organic no mechanical explanation has yet been given. The universal applicability of this type of explanation cannot therefore be considered more than a methodological ideal. The teleologist appears to occupy an analogous position. He moves with ease in the organic realm, although even here he cannot yet maintain that his task is completed; but when he enters the realm of the inorganic, he is unable to proceed at all unless he adopts some elaborate and unverifiable hypothesis about the nature of matter. Thus, the obvious fact is that phenomena which can readily be given a mechanical explanation are such as afford little apparent ground for a teleological explanation, and vice versa.

Suppose now that all phenomena whatsoever could be divided into three classes as follows: (1) phenomena of which there exists a mechanical but not a teleological explanation, (2) phenomena of which there exists a teleological but not a mechanical explanation, and (3) phenomena of which there exists neither a mechanical nor a teleological explanation. In such a situation, the chance of a conflict between the mechanist and the teleologist would appear to be remote. Of course the mechanist might maintain that present-day teleological explanation is illusory, and that future knowledge will make it clear that the phenomena in classes (2) and (3) can be explained mechanically. And if the teleologist were to adopt an analogous position with regard to present-day mechanical explanations and the phenomena in classes (1) and (3), a conflict certainly would result. Such a conflict, however, would be based upon faith in the universal applicability of the methods concerned, and, in the absence of evidence, would be unreasonable. Hence, the conflict that now arises is not based upon such faith. The fact is that the above supposition is not true, there being a fourth class of phenomena, those, namely, of which explanations of both kinds are offered. This brings the trouble to a definite head. For of these

phenomena the teleologist maintains the correctness of his own and the falsity of the mechanical explanation, while the mechanist similarly asserts that his explanation is the only true one.

The coloration of the humming-bird, for instance, has been explained both teleologically and mechanically. Teleology urges that the phenomenon is explained by its function. This function, it says, is sexual attraction, as a result of which the survival of those birds that were colored in special ways was guaranteed. The mechanist points out, however, that the humming-bird is normally in incessant motion, and urges that the relatively large quantities of waste products accumulating in its feathers as a result of its activity is the correct explanation of its coloration.

Now, in so far as the teleologist denies that the mechanical, and the mechanist that the teleological explanation, is true, there is a definite conflict between them; but it is a conflict which investigation, theoretically at least, could remove. A carefully performed investigation might show that the coloration of the humming-bird subserves no biological purpose, or it might prove that the mechanical explanation is incorrect. The important point, however, is that it could, theoretically, prove that both explanations *are true*. For these explanations, so far from being logical contradictories, are not even logical contraries. In themselves they do not conflict at all. The only possible conflict occurs when the advocate of one explanation denies the other. And this is unjustifiable. It might quite well be that the coloration of the humming-bird is caused as the mechanical explanation asserts, or at least in some rather similar way, and that it has had a biological significance. The one explanation states that the phenomenon has a cause; the other, so far from denying this, merely asserts that it has a function. No conflict is possible between two theories one of which states that a certain phenomenon has a cause and the other that it has an effect.

The harmony between the two explanations is rendered clearer by considering what "natural selection," which is the essence of much teleological explanation, really is. Darwin accepted organic variations as one of his ultimates, and he then endeavored to show that, with variations in the environment (in the widest sense of the term), one organic variation survives rather than another. He did not deny that organic variations have causes, although he professed himself entirely ignorant as to what these causes are. Now with regard to such a phenomenon as the coloration of the humming-

bird, the teleologist really adopts the Darwinian hypothesis in its original form, while the mechanist has proceeded, not to deny this hypothesis, but rather to enlarge it by assigning a cause for one of the organic variations.[2]

The conflict between the mechanist and the teleologist nevertheless continues, and its most acute stage is reached when both put forward explanations of religious phenomena. Let us suppose that it be asked why a certain man prays. The teleologist, having regard to the function of prayer in the man's life, may say that it is because prayer uplifts and strengthens him. If he answers thus, he is giving what is formally a perfectly good teleological explanation, which may also be true in fact. But the mechanist may say that a man prays because his mother has taught him to pray. Is there then a conflict between the mechanist and the teleologist? Not if "because" be used in an appropriate sense by each. For it seems unquestionable that both explanations may be correct: the man may be uplifted and strengthened by prayer and he may pray now because he has been taught to pray. While this cannot very well be denied, the teleologist usually exhibits a certain hostility toward the mechanical explanation. This hostility increases in proportion as the phenomena concerned become more extensive, reaching a climax when such phenomena become synonymous with religion as a whole. The situation, however, is clear, and there is no ground whatever for the hostility. Just as the coloration of the humming-bird may have both a mechanical and a teleological explanation, so may any phenomenon whatsoever. If therefore the teleologist is interested in the function of religion, in the purpose it plays in social and individual life, he may limit his statements to the making explicit of this aspect of religion, and is quite justified in so doing. But the man who seeks for the causal antecedents of religion is equally justified, and his mechanical explanation is in no way opposed to that of the teleologist. The one type of explanation is without prejudice to the other.

The most that an advocate of one type can say to an advocate of the other is that he is only interested in his own type of explanation. Such an attitude, however, would be onesided. An impartial and complete review of religion will include a consideration both of its function and of its antecedent conditions. Indeed, as

[2] By assigning a particular kind of cause, however, namely, a conscious purpose, it is often thought that a teleological explanation of organic variations is given; but see below, with regard to the explanation of a man's possession of a boat.

we are able to control events only in so far as we know their causes, the mechanical explanation of religion seems to be as valuable as, if not more valuable than, its teleological explanation. And there is a further reason for the importance of mechanical explanations of religious phenomena at the present time. In the past, such explanations have often involved too much hypothesis about the beginnings of mental life to render them either convincing or useful, not to mention that they have usually laid themselves open to the charge of merely reconstructing complex evolved phenomena out of elements that are themselves products of evolution. What is now needed is a treatment of religious phenomena which shall connect them with psychical processes such as we know them. Such a treatment would distinguish between a man's psychical constitution and what tradition gives him. With this distinction in mind, it would explain the life of a present-day religious man by reference to *his* psychology and not by reference to that of some primitive ancestor. And it would endeavor to indicate the *psychical tendencies* which lead the religious man *to accept the religious tradition.*

It is fairly obvious that the easy adoption of this standpoint requires a recognition of the fact that mechanical and teleological explanations, so far from being in opposition, are complementary. In the past, various obstacles have stood in the way of this recognition. It will therefore be of use to consider now what the more important of these obstacles are.

A reference may first of all be made to the influence of rash statement. It is always difficult for the enthusiast to keep his speech within the bounds of logic; but when the mechanist, for example, asserts positively, and in the absence of anything that could be called strict scientific evidence, that the cause of religious experience is matter in motion, he inevitably finds himself in conflict both with the sober teleologist and with the teleologist who is as rash as himself. This, however, is obvious; and since the conflict thus arising is not in itself any reason why the complementary character of the two types of explanation should not be recognized, it is unnecessary to consider it further.

We now come to a more important point. Some confusion seems frequently to have been due to the employment of the term "explanation" without any precise definition of its meaning. It is commonly thought that an explanation is a complete account of a thing, so that if one explanation is true, any other must be false.

This no doubt is only felt in a vague kind of way; and it would, in fact, be an error to say that the philosophical explanation of a thing consists in stating all the propositions that are true of it. All explanation is the establishing of certain kinds of connection between the phenomena to be explained and others; and no explanation includes the proposition that it is a complete explanation. Possibly, it would be an advantage if the term "explanation" were not to be used in the present connection. If it were to be definitely recognized that a mechanical "explanation" consists in assigning causal antecedents, while a teleological "explanation" is the assignment of function, it would be realized that the two types of "explanation" do not conflict at all, but that they are, in fact, a viewing of phenomena from two standpoints.

Apart from this difficulty concerning the term "explanation," there has been a perhaps greater difficulty in connection with the term "teleology." There has as a rule been no clear conception of what was being done in giving a teleological explanation. Perhaps the chief point of confusion here has been in the idea that a teleological explanation of any phenomenon consisted in showing that there existed antecedent to the phenomenon a purpose in some mind, and that this purpose brought the phenomenon into being. Let us suppose, for example, that a man who lives on the bank of a river is asked why he has a boat. He may reply that he has it for pleasure. In giving such a reply, he would be offering a good teleological explanation of his possession of a boat,—he would be explaining this fact by the function which the boat has in his life. But he might conceivably have given a mechanical explanation. He might have replied that he has the boat because a friend one evening suggested he should procure one, that the idea struck his fancy, that he happened to possess the necessary money to buy one, that there already existed a boat which could be bought, and so on. Such an explanation would certainly be as true as any teleological explanation. If the boat did give the man pleasure, his possession of it on the other hand certainly had causal antecedents. That is to say, his possession of the boat had both a cause and an effect. And so far as the mechanist restricts himself to the assignment of a cause and the teleologist restricts himself to the assignment of a function, no conflict is possible.

But now there frequently arises a misapprehension. Among the causal antecedents of the man's possession of the boat, there

was a certain state of his mind, which included a purpose. It might be maintained that of all the causal antecedents this was the most important and deserves to be specially emphasized in any explanation of the man's possession of the boat. And it must be admitted that this state of the man's mind *was* important, although whether it was more important than the fact that he possessed a certain amount of money, is not easy to determine. But however important it may have been, it must be noted that it is a part of the mechanical explanation of the fact to be explained: it is one of the fact's causal antecedents.[8] Now the teleologist seems to have frequently held that this factor constitutes the teleological explanation of the fact. He sees that the conscious purpose is important; and he appears to confuse a conscious purpose which is an antecedent cause of a phenomenon, and therefore part of its mechanical explanation, with the "purpose" of the phenomenon in being a means to an end. "Purpose," in the latter sense, however, does not refer to a conscious state, but merely to the manner in which a thing functions,—to what might be called, in a broad sense, an effect of the thing.

That a conflict should have arisen between the mechanist and the teleologist as a result of such a confusion was inevitable. It frequently led to the teleologist selecting *a part* of the mechanical as *the* teleological explanation; and his conflict with the mechanist has consequently often been due to the fact that what is admittedly an important part of the mechanical explanation is asserted by the teleologist to be the whole explanation and to be teleological. The teleologist did not see that his explanation was really mechanical, nor that it was precisely this which caused the conflict between himself and the mechanist. The two kinds of explanation could not be regarded as complementary while the teleologist asserted one antecedent cause and the mechanist another. In fact, the conflict between mechanist and teleologist has often been essentially similar to a dispute which would arise between two scientists were one to say that the *cause* of a certain event is A, and the other that this *cause* is B, or at least A plus C.

Connected with this misapprehension of the true character of teleological explanation, is the more or less popular view of the divine ordination of religion. According to this view, the true explanation of religion, which as a result of the above confusion

[8] The psycho-physical parallelist would deny this; but, then, *he* could not hold that the man's purpose was of the slightest importance, as it, for him, could not affect *any physical* phenomenon.

is regarded as teleological, is that a purpose in God's mind determined its existence. This is, of course, a mechanical explanation. And it conflicts with what is ordinarily called mechanical explanation of religion because this is usually, in its aim, scientific,—in that it attempts to discover *verae causae*. In so far, therefore, as such a "teleological" explanation was implicitly accepted, it would produce hostility toward such mechanical explanation as endeavored to be scientific. The conflict thus arising would really be between a scientific and an unscientific mechanical theory.

The misapprehension just considered leads to the position that any given "teleological" explanation must conflict with any given mechanical explanation. But apart from such *post facto* conflict, the teleologist, as has been noted above, frequently objects to the very idea of mechanical explanation. It appears to me that the reason of this attitude is largely to be found in two ideas, which must be considered briefly.

The first of these concerns the causal relation. Among the various theories with regard to the nature of this relation there has appeared the view that cause and effect are identical, the effect being merely a transformation of the cause. Such a theory is not compatible with certain common usages of causal terminology; but, on the other hand, it appears to be implicit in much popular belief. Now a person who accepted such a theory would be disinclined to admit that something regarded by him as valuable had a cause less valuable than itself. As it is generally considered that superstition in its more primitive forms and the feelings associated with it are less valuable than enlightened religion, a certain hostility arises toward any attempt to show that the former is at least part of the cause of the latter. Since this, however, cannot be very well denied, the teleologist admits it with a somewhat bad grace, maintaining that the assignment of causal antecedents is not *explanation*. Such an assertion has no point once it is seen what explanation is.

It is unnecessary to enter here upon any criticism of this conception of the causal relation, although it could be shown not to be in agreement with recent conclusions on the subject. The important point is to maintain that the value of a thing is independent of its genesis. If an institution, for instance, really is valuable at the present day, it remains valuable whatever its origin may have been. To say that its value is depreciated by supposing that its cause was less valuable than itself is to admit that it is not *it* that is valued

but some belief about it. The value then naturally disappears when the belief is proved to be false. It is therefore unreasonable to depreciate the value of religion, for example, because of the undoubted fact that superstition moulded its earlier forms. Indeed, it would be more reasonable for the man who is convinced of the value of religion to place upon earlier superstitions a higher value; for have not they been part of thc cause of this valuable thing? There is, however, a strong disinclination to adopt this point of view. In spite of anthropological evidence, the teleologist will almost deny that religion could have an origin with which evil was intimately associated. It is probably here that we should look for the explanation of the not uncommon attitude of hostility toward such theories of religion as were propounded by Holbach and the Encyclopedists. Of course, objection may be taken to such theories on the ground that they assign causes wrongly; but to him who considers religion valuable, no reasonable objection to them can be made to depend upon the fact that the causes assigned would be considered by us destitute of value, or even positively bad.

Perhaps an even more important factor in the teleologist's hostility toward the mechanical explanation of religion is the idea, often felt more or less dimly rather than clearly cognized, that any account of the genesis of an institution tends to lessen its authority over the individual. Those who believe that the authority exercised by religion is productive of excellent results, naturally desire that the authority of religion shall not be weakened. The teleologist, who often holds this belief, therefore tends unconsciously to object to *any* mechanical explanation of religion. This tendency is perhaps strengthened by a kind of unconscious pragmatism, by the almost unconscious belief that a theory that has bad consequences must be false. There thus arises on the part of the teleologist a hostility to the mere form of mechanical explanation, a hostility which is rendered greater by a great hope and a great fear.

The question involved here must be admitted to be important. But there appears to have been some misapprehension as to what this question is. The situation is not such as to justify hostility toward explanation merely because it is mechanical in form. To suppose that it is, is to miss the point entirely. The question belongs, in fact, to the sphere of practice rather than to that of theory, and may be termed in a general sense educational. From a practical standpoint, it can be inquired whether it is justifiable to propagate

a knowledge of the genesis of certain institutions if it seems likely that such knowledge will have bad consequences; but the question, what is the truth, is quite distinct from the question, should the truth be freely given to all. It may be practically best at a given period to prevent the knowledge of the genesis of religion, for example, (supposing that we possess it), from becoming popular; but this is by no means equivalent to admitting that religion has no antecedent cause. If, therefore, the teleologist here makes an objection to the mechanical theory, he must base it on the unwisdom of the mechanist in publishing his theory and not on that theory's form.

Such appear to be the chief factors in the mechanical explanation of the belief that teleological and mechanical theories of religious phenomena are absolutely opposed to each other. Contrary to this belief, our general conclusion must be that the two forms of explanation are complementary and in no sort of conflict. It is perhaps necessary to add that by "mechanical explanation" here is not meant "explanation by matter in motion exclusively" (*vide* first paragraph of article).

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AN EMPIRICAL VIEW OF THE TRINITY.

There are many rationalistically-minded theists to-day who wonder how intelligent persons can continue to use the language of the old Trinitarian dogma. God to them is an unquestioned reality; although we never see or hear Him, and cannot clearly say where or how He exists, we can be sure that He does exist. But that He is One God in Three Persons seems to them utterly unintelligible and a remnant of scholastic metaphysics which modern common sense should repudiate. Surely, when Christianity is thoroughly rationalized, this incomprehensible and self-contradictory doctrine must yield to the clean-cut Unitarian conception.

In answer to this familiar contention, it would not be a paradox to say that the mystical Trinitarian formula, though, to be sure, it is clothed in the creeds with an unwarranted license of language, is based far more firmly upon experience than the more sharply defined theistic conception of the rationalists.

UMI

For how do we know, after all, that God exists? The old naive faith in the Biblical legends of a Jehovah who walked in the garden in the cool of the afternoon, who conversed with the saints, and wrote the Decalog with his finger upon tables of stone, is obsolescent. We can no longer believe in the existence of such a person just because the writers of the Bible-documents believed in him, any more than we can believe in Apollo or Artemis because the Greek epic poets believed in them. We cannot even believe in him because Jesus believed in him; a Jew of the first century, steeped in the pious hopes of his countrymen, how could he possibly help believing? His genius was religious, not scientific; he was no analyst, as Phillips Brooks once said; the whole bent of his nature led him to adopt the faith of his fathers, deepening, sweetening, spiritualizing it, but certainly never questioning its essential truth.

Nor can we rely any longer upon the stock arguments of the older theology—arguments from design, first-cause, *et al.* Every one of these has been so riddled with objections, or had its fallacies so exposed, that it needs an unread or obtuse theologian to rely upon them. The younger generation leaves the dust to gather upon all this laborious argumentation, pro and con, and turns to religious experience as the sole source of its faith and hopes. The question that now engages attention is, How is God revealed in human experience? Psychology and biography take the place of metaphysics, introspection and observation of *a priori* reasoning. The conviction is growing that the conception of God does not rest upon inferences from the nature of the universe, and still less upon a supernatural revelation, but upon the concrete facts of the religious life. God-experiences (if we may use the phrase) are primary, God-theories are secondary. So that even if our theorizing remains dubious and confused, these experiences are indisputable and precious; even if we were to discard the term "God" entirely, the Reality which we seek to name thereby would remain, of profound importance in the religious life of man.

And now, from historical and phychological studies this conclusion emerges: our experience of God is of three types—there are three sorts of human experience which, together, give us our conception of God.

Historically, the concept of God came into existence in these three principal ways: it was the crystallization of the awe and reverence and fear and faith felt in the contemplation of nature, felt

in the thought of deceased heroes or chiefs believed to be still alive, felt in the response to that inward pressure that we call conscience. We have passed beyond the stage of disputing whether religion had its origin in animism or in the belief in ghosts; it sprang from both sources. And—what is less commonly recognized—it sprang from the inner struggles of those prehistoric ancestors of ours who, millenniums before St. Paul, found two natures battling within them, the one devilish, the other divine.

And what is true of its origin is true of our God-idea still. In its fulness it is formed by the convergence of three great streams of mental tendency—the recognition of the Divine in nature, in our spiritual heroes, and in ourselves. God about and beyond us, in the vastness of the cosmic life; God in whatever religious leader the believer follows, the spiritual power in that other human life upon which he leans for guidance and inspiration; God in his heart, the Holy Spirit in him, to which he must give his entire allegiance if he would find lasting satisfaction and peace,—man's conception of God is naturally Trinitarian. For the Christian, Christ is pre-eminent among spiritual heroes, epitomizing and typifying that Divinity in other men which is our greatest source of salvation; he is the supreme revealer of God to us, the symbol and concrete incarnation of Godliness in man. And so the Trinitarianism of Christianity, derived as it was by a devious and blind process of intuition and easily refutable dialectic, has not been, after all, a wide departure from man's spontaneus and instinctive reactions to the great and mysterious forces without him and within.

The belief in God is not, then, a mere act of credulity, a venture of faith in the unknown, an "over-belief," sufficient for our personal needs but unverifiable, unprovable to others. Such an adventurous belief might indeed be legitimate; but is it all we can have? No. The concept of God is, in its foundation-sense, empirical; it is not, at the outset, a matter of blindly believing, but of opening our eyes to see. It is one of Matthew Arnold's greatest services to thought that he insisted upon this truth. We may not deem his definition of God, as "the Power not ourselves that makes for righteousness," comprehensive enough; but we must applaud his attempt to point out in the conception of God, so largely being discarded as a mere superstition, a substratum of truth.

For there surely is in the world a great current pushing us into righteousness. In struggling to do right we are not setting

up merely arbitrary and conventional standards, we are moving in the direction which the forces of Nature have ordained. No matter how men may rebel and kick against the pricks, morality is bound eventually to win the day. In choosing virtue we choose the winning side; the cosmos is backing us. In this knowledge there is inspiration and assurance.

Moreover, the same cosmic process that has, from the beginning, been moving irresistibly in the direction of producing, in due time, human virtue and valor, has also flowered out into beauty and all other forms of good. Forces making for evil there also are, for ugliness and sorrow and sin; and men have widely differed in their reaction to that truth, some forming from it a devil-concept, some clinging to the faith that it contributes in the end to good, some ignoring it, throwing it overboard, as the mere waste-product of life. But to realize through every fiber of our being the presence of the Power for Good, the God-Power, as enduring and ultimately winning, to pledge our individual efforts on its side, and rejoice in its triumphs, is the essential *differentia* of the religious life.

The great seers and saints have realized more vividly than the average man the presence of this God in nature. From the Psalmists to Wordsworth, Carlyle, Emerson, we find men of vision inspired and consoled by the sight of this tide of Good that sweeps man on to a destiny which he but dimly sees. Much that was superstition and error has been mingled, no doubt, with this vision. This God whose glory the heavens declare was deemed a partisan Jewish deity, with a manlike form and speech, offering a crude extraneous reward and punishment to those who followed or disobeyed his will. But such anthropomorphism is better than an absence of vision. For rewards and punishments for virtue and vice there are, though they are intrinsic, and brought about in natural ways. And to lose the sense of the divineness of nature, to lose the faculty of worship, of reverence, of joy in the beauty and wonder of the world in which we live, is not only to be, in so far, irreligious, but therein to miss one of the essential ingredients in the noblest and happiest life.

But more than in nature do we find God in men—the best men we know; and for us at least of Christendom, in Christ. This need not imply any disloyalty to a truly historic conception of his life and teachings, need not imply anything miraculous or supernatural about him. The divineness in Christ may be as much a natural fact,

produced according to natural laws, as the divineness in the outer world may be. For our purpose here we need not debate that question. For certainly it was not the debated fact of supernaturalness (in the scientific sense) that made Christ divine; it was his character. His will was wholly merged with the will of God; there was no selfishness, no self-indulgence, in him. The Christ-life is the divine life for men, the measure of the amount of Godliness that we are capable of. To call his life divine is not in the least to assert that he was born of a virgin, raised the dead, rose himself from the tomb; it is a different sort of judgment, a value-judgement. The facts about his life must be decided by historical methods, as we would sift the records of the life of any other personage of the past; no ardent believer or entrenched ecclesiasticism ought to attempt to bias the impartial judgment of scholars upon them. But the question of the divineness of this life is to be decided by men of spiritual vision. And the verdict of truly religious men is all but unanimous; the great warrior, the great statesman, the great inventor, the great poet, have a veritable spark of God in them; but the life that is most truly divine, that most fully reaches up to God, is the life of purity and charity and self-sacrifice. Preeminent among such lives, dazzling men of all races for the two millenniums since he lived, is the life of the Carpenter of Nazareth.

But if divinity is especially incarnate in the spiritual heroes of mankind, it may also, in some measure live in each of us. We recognize amid the tangle and clutter of selfish and sensual desires a holier spirit within us. Sometimes an uprush of noble feeling or high resolve, a power for good, wells up in us, and we know it to be divine. This fountain of inner holiness springs up at times abruptly, even unexpectedly, and then oftenest subdues our other impulses into hushed obedience. But we need not consider these conversion-experiences as supernatural; the new spirit is holy, not because of its miraculous way of working, but because its influence in our lives is divine. The practically significant fact is that this power is ready for our use. As Emerson wrote, "It is a secret which every intelligent man quickly learns, that beyond the energy of his possessed and conscious intellect, he is capable of a new energy (as of an intellect doubled on itself) by abandonment to the nature of things; that besides his power as an individual man, there is a great public power upon which he can draw, by unlocking at all

risks his human doors, and suffering the ethereal tides to roll and circulate through him."

What all this may imply about the ultimate metaphysical nature of God is, no doubt, worth discussing, and conceivably of great import. But men are coming in these latter days to a humbler sense of their intellectual limitations; we are realizing that we know nothing of the inner nature of anything, save of our own conscious life as it passes. What is matter? What is electricity? What is God? Perhaps we cannot know. But what is practically important is to understand and utilize the experiences out of which these concepts have grown. If we can use electricity in our telephones and dynamos and trolley cars, we can be content to confess our ignorance of its inner nature. So if we can comprehend and repeat the religious experiences out of which the concept of God has arisen, it matters less if our knowledge of God is limited to that experience-contact.

Souls of different types and needs will naturally formulate their experience in different terms; there is no need that any one to whom the generalization is not personally useful should express his God-idea in a trinitarian formula. Trinitarianism should never be a dogma. And with the arguments and disputes of the Greek doctors of the third and fourth centuries, through whom that dogma took shape, we may have scant patience. Certainly all that sort of speculation is very alien to our modern scientific world-view. But on the other hand, the arguments of the rationalists of to-day for a God-idea divorced from those experiences in which it has its natural roots, are equally alien to the outlook and spirit of science. To believe in God is a mere act of credulity, except as we see the meaning of the God-idea in human life. When we do thus turn to experience, we find ourselves led to the God-conception from the three sorts of experience mentioned. So, as an embodiment of the profound truth of the threefold basis of our human conception of God, the Trinitarianism of the saints should command our sympathy and respect.

Trinitarianism, Unitarianism—as mutually exclusive dogmas, both are cramping and arrogant. What is important is to keep alive the experience that each term enshrines. The essential oneness of all God-experiences, and of the God-idea which they unite in producing, is important, no doubt. But the bare insistence upon unity has, now that the extravagances of polytheism are forever

past, little religious value, and tends to a contentment with less than the full gamut of religious experience. No one of the three forms of God-experience can be dispensed with in a rich and fruitful spiritual life; and it is no wonder that the orthodox have generally felt a merely negative Unitarianism to have an impoverishing tendency. However crude the creedal affirmations of Trinitarianism may be, the fulness of the Christian life has by it been fostered and preserved. So, however loath we may be to seem to accept the description of a quasi-human Being who is somehow Three Persons and yet One, if we take the doctrine (as we must take all religious doctrines) in its inner and spiritual sense, which is its empirical foundation-sense, we shall see it as a more or less blind expression of a great truth—that Christians can attain to the vision of God in three ways, through contemplation of the outer world, through faith in their Master Christ, and through obedience to the Holy Spirit in their hearts.

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GENERAL NOTES ON THE CONSTRUCTION OF MAGIC SQUARES AND CUBES WITH PRIME NUMBERS.

The series of numbers generally used in the construction of magic squares are in arithmetical progression. The progression of the prime number series is very irregular, and therefore cannot be used as freely as an arithmetical series. This naturally leads to the investigation of the possible irregularities in groups or series of numbers which may be formed into magic squares. It is also necessary to find means of discovering these groups of numbers in the prime number series.

It is the writer's aim to describe here simple rules for constructing prime number squares, methods of finding the numbers to be used, and to point the way to the solution of a few of the problems not yet mastered.

THE SQUARE OF THE 3d ORDER.

There is only one possible construction of this square and there is only one rule governing the series, and that is, when the series is written in tabular form, as in Fig. 1, the differences between all

vertically adjacent cells must be equal, and the differences between all horizontally adjacent cells must be equal, but the vertical and horizontal differences must be unequal to avoid duplicate numbers. These differences are indicated by numbers at the sides of the lattice and it will be by these that we will identify the nature of the series

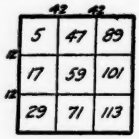


Fig. 1.

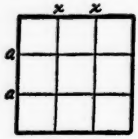


Fig. 2.

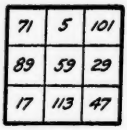


Fig. 3.

used in the following magic squares. We will represent these differences by letters, using the letters of the fore part of the alphabet for one set of differences and those of the other end of the alphabet for the other set, as is shown in Fig. 2, like letters indicating the necessity of like differences.

Fig. 2 is arranged into the magic by using the middle column and middle line as diagonals, the position of the remaining numbers then being easily found. The resulting magic is shown in Fig. 3.

THE SQUARE OF THE 4th ORDER.

Any series or set of 16 numbers, when written in the tabular form previously mentioned, wihch gives the differences a, b, c and x, y, z, may be formed into a magic square by the Jaina method as follows. Fig. 4 shows a table of prime numbers with irregular differences. Four sets of the upper line of numbers of this table are arranged

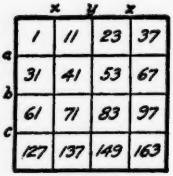


Fig. 4.

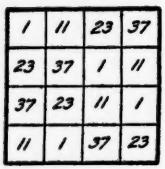


Fig. 5.

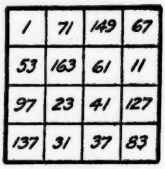


Fig. 6.

in a subsidiary square, as shown in Fig. 5, so that each line, column, and the two diagonals contain each of the four different numbers. Subtracting the initial number of the table (in this case 1) from each of the numbers in the left-hand column of the table, will give the numbers, 0, 30, 60, 126, which are to be arranged in a second subsidiary square with the same arrangement as in Fig. 5, only that

the pattern is turned 90 degrees. The two subsidiary squares are then added together, cell to cell, to produce the magic square. A resulting square is shown in Fig. 6.

In selecting numbers from the tables for the subsidiary squares, the column and line containing the lowest numbers should be chosen, but it makes no difference which set the initial number is subtracted from.

A balanced series of numbers whose tabular differences are

x	y	x	x
a	17	47	137	167
b	29	59	149	179
a	31	61	151	181
	43	73	163	193

Fig. 7.

193	47	137	43
29	151	61	179
31	149	59	181
167	73	163	17

Fig. 8.

17	73	149	181
179	151	47	43
61	29	193	137
163	167	31	59

Fig. 9.

a, b, a, and *x, y, x,* may be arranged into an *associated* square, or a *pandiagonal* square. Such a series is shown in Fig. 7. By revolving the two diagonals of Fig. 7, 180 degrees, it will produce the associated magic square shown in Fig. 8. To produce a pandiagonal square, we select, as before mentioned, two subsidiary sets of numbers from which are formed two subsidiary squares of the pattern shown in Fig. 5. The numbers in the upper line should be so arranged that the sum of the left-hand pair equals the sum of the right-hand pair. One of these subsidiary squares is revolved 90 degrees and added to the other to produce the magic. A pandiagonal square resulting from such a construction is shown in Fig. 9.

Another form of subsidiary square which may be used to produce a pandiagonal square from a balanced series is shown in Fig. 10, which is exemplified with arbitrary numbers. The numbers

1	4	1	4
2	3	2	3
4	1	4	1
3	2	3	2

Fig. 10.

167	163	17	73
29	61	179	151
193	137	43	47
31	59	181	149

Fig. 11.

must be so arranged that the pairs indicated by dotted lines will have like summations. One subsidiary square is revolved 90 degrees from the other and the two added together to produce the final

square. Fig. 11 is a pandiagonal square produced from the series in Fig. 7, by the method last described.

THE SQUARE OF THE 5th ORDER.

A series of 25 numbers whose tabular differences are a, b, c, d and w, x, y, z is shown in Fig. 12. Such a series may be formed

	w	x	y	z
a	1	7	61	67	127
b	13	19	73	79	139
c	23	29	83	89	149
d	37	43	97	103	163
	41	47	101	107	167

Fig. 12.

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

Fig. 13.

1	29	101	79	163
73	103	127	23	47
149	41	19	97	67
43	61	89	167	13
107	139	37	7	83

Fig. 14.

into a *pandiagonal* square as follows. Two sets of subsidiary numbers are selected and arranged in two subsidiary squares according to the pattern shown in Fig. 13; the pattern of one subsidiary square should, however, be in a reversed or reflected order from the other. The two squares are then united to form the final square. Fig. 14 shows one example resulting from the series in Fig. 12.

To produce an *associated pandiagonal* square of the 5th order, it requires a series whose tabular differences are a, b, b, a and x, y, y, x. The writer, at present, knows of only one series which suits the above requirements. Its initial number is 41 and the tabular differences are 60, 390, 390, 60 and 72, 138, 138, 72 respec-

1013	251	449	911	881
839	1301	941	113	311
41	173	701	1229	1361
1091	1289	461	101	563
521	491	953	1151	389

Fig. 15.

tively. The subsidiary squares are arranged in associated formation and according to the pattern in Fig. 13. The solution of this difficult problem was accomplished by Mr. Chas. D. Shuldham, and his resulting magic is shown in Fig. 15.

Mr. Shuldham has succeeded by other methods in constructing

associated squares of the various orders including the 12th.[1] These are not pandiagonal however.

THE SQUARE OF THE 6th ORDER.

A balanced series whose tabular differences are a, b, c, b, a and

	x	y	z	y	x
a	47	179	199	1301	1321	1453
b	257	389	409	1511	1531	1663
c	467	599	619	1721	1741	1873
b	677	809	829	1931	1951	2083
a	887	1019	1039	2141	2161	2293
	1097	1229	1249	2351	2371	2503

Fig. 16.

1	6	6	6	1	1
5	2	5	2	2	5
3	4	3	3	4	4
4	3	4	4	3	3
2	5	2	5	5	2
6	1	1	1	6	6

Fig. 17.

x, y, z, y, x may, by a common method, be formed into a magic square as follows. Fig. 16 shows a series with the above differences. It will be noted that the values for a, b and c are each 210, though the method of construction does not necessitate these like differences. Subsidiary sets of numbers are selected as previously explained and formed into subsidiary squares according to the pattern shown in Fig. 17. One of the squares thus formed is revolved 90 degrees and added to the other, cell to cell, to form the final square. One

47	2293	2083	1873	257	1097
1321	389	1951	599	1019	2371
1249	1511	619	829	2141	1301
2351	1039	1721	1931	409	199
1229	1531	809	1741	2161	179
1453	887	467	677	1663	2503

Fig. 18.

	v	w	x	y	z
a	19	41	47	67	101	173
a	229	251	257	277	311	383
2a	439	461	467	487	521	593
a	859	881	887	907	941	1013
a	1069	1091	1097	1117	1151	1223
	1279	1301	1307	1327	1361	1433

Fig. 19.

example is shown in Fig. 18, which is magic only in its lines, columns and two diagonals.

A series of prime numbers to suit the above differences is very difficult to find, but the following form of series is easily found, and it is only recently that the writer has succeeded in discovering a

[1] See "Associated Prime Number Magic Squares," *The Monist*, July, 1914, Vol. XXIV, No. 3.

method of arranging the numbers of such a series in magic square formation.

Fig. 19 shows a series whose tabular differences are *a, a, 2a, a, a* and *v, w, x, y, z*. The subsidiary numbers with differences corresponding to the *a* values are arranged as shown in Fig. 20, each

1260	1260	1260	0	0	0
210	210	210	1050	1050	1050
420	420	420	840	840	840
210	210	210	1050	1050	1050
1260	1260	1260	0	0	0
420	420	420	840	840	840

Fig. 20.

19	41	47	67	101	173
101	67	19	173	47	41
41	47	173	19	67	101
47	173	41	101	19	67
173	101	67	47	41	19
67	19	101	41	173	47

Fig. 21.

line made up of three like pairs. The other set of subsidiary numbers is arranged as shown in Fig. 21, with a full set of the six different numbers in each line, column, the two central diagonals, and in the cells corresponding to the like numbers in Fig. 20. These two subsidiary squares combined, will produce the magic square shown in Fig. 22.

1279	1301	1307	67	101	173
311	277	229	1223	1097	1091
461	467	593	853	907	941
257	383	251	1151	1069	1117
1433	1361	1327	47	41	19
487	439	521	881	1013	887

Fig. 22.

To construct an *associated* or *pandiagonal* square of the 6th order would require a table of numbers with the differences *a, b, c, b, a* and *x, y, z, y, x* and each set of subsidiary numbers would have to be of values permitting thier arrangement in magic 2×3 rectangles.[2] It is doubtful if a series of this nature can be found, but, by methods previously published, the problem may be solved

[2] To construct, see "Notes on the Construction of Magic Squares" by Messrs. Andrews and Frierson, *The Monist*, April 1912, Vol. XXII, No. 2. For an example, try any series with the tabular differences 2, 1, 4, 1, 2 and 9, 11, 18, 11, 9, with the illustrations on p. 306 (of the above issue) as a guide.

by first forming an *associated* square and transforming it into a *pandiagonal* square.[3]

THE SQUARE OF THE 7th ORDER.

This square, like the 5th order square, may be formed into a pandiagonal magic by using a series having tabular differences of

	s	t	v	w	x	y	z
a	17	41	173	347	613	853	1997	2693
b	23	47	179	353	619	859	2003	2699
c	227	251	383	557	823	1063	2207	2903
d	233	257	389	563	829	1069	2213	2909
e	443	467	599	773	1039	1279	2423	3119
g	653	677	809	983	1249	1489	2633	3329
h	857	881	1013	1187	1453	1693	2837	3533
	863	887	1019	1193	1459	1699	2843	3539

Fig. 23.

a, b, c, d, e, g and *t, v, w, x, y, z*. The subsidiary squares have like numbers running in knight paths, the pattern of one may be a reflection of the other, or they may have the same pattern, but 90 degrees apart.

THE SQUARE OF THE 8th ORDER.

This square may be formed with an irregular series having tabular differences of *a, b, c, d, e, g, h* and *s, t, v, w, x, y, z*. One series is shown in Fig. 23, and Fig. 24 shows one pattern of sub-

1	2	5	6	3	4	7	8
4	3	8	7	2	1	6	5
2	1	6	5	4	3	8	7
3	4	7	8	1	2	5	6
5	6	1	2	7	8	3	4
8	7	4	3	6	5	2	1
6	5	2	1	8	7	4	3
7	8	3	4	5	6	1	2

Fig. 24.

17	257	619	1063	599	1193	2633	3533
353	383	2693	2213	677	857	1279	1439
467	863	1489	1453	347	389	2699	2207
809	1187	2423	3539	23	251	613	1069
823	859	233	41	2837	3329	1019	773
2909	1997	557	179	1699	1039	881	653
1693	1249	887	449	2903	2003	563	173
2843	3119	1013	983	829	853	227	47

Fig. 25.

[3] See "Pandiagonal Prime Number Magic Squares," by Mr. Chas. D. Shuldham, *The Monist*, Oct. 1914, Vol. XXIV, No. 4.

sidiary square, other patterns being easily found. The magic square shown in Fig. 25 was constructed from the above series with the subsidiary squares arranged according to Fig. 24, one being revolved 90 degrees from the other.

THE SQUARE OF THE 9th ORDER.

It will be noted in Fig. 24 that each line, column and the two diagonals contain no like numbers. The numbers are also arranged

	r	s	t	v	w	x	y	z
a	17	41	173	347	613	853	1997	4871	5431
b	23	47	179	353	619	859	2003	4877	5437
c	227	251	383	557	823	1063	2207	5081	5641
d	233	257	389	563	829	1069	2213	5087	5647
e	443	467	599	773	1039	1279	2423	5297	5857
f	653	677	809	983	1249	1489	2633	5507	6067
h	857	881	1013	1187	1453	1693	2837	5711	6271
k	863	887	1019	1193	1459	1699	2843	5717	6277
	1277	1301	1433	1607	1873	2113	3257	6131	6691

Fig. 26.

so that the pattern will set upon itself, by reflection or revolving, and not produce duplicate numbers in the final square. In like manner, the subsidiary squares for the 9th order square are ar-

3	8	4	7	6	2	5	1	9
7	6	2	5	1	9	3	8	4
5	1	9	3	8	4	7	6	2
4	3	8	2	7	6	9	5	1
2	7	6	9	5	1	4	3	8
9	5	1	4	3	8	2	7	6
8	4	3	6	2	7	1	9	5
6	2	7	1	9	5	8	4	3
1	9	5	8	4	3	6	2	7

Fig. 27.

5641	863	829	1187	5507	179	467	853	3257
881	1489	2003	5857	17	1873	557	5717	389
773	4871	1433	251	1699	2213	6271	653	619
233	823	6277	4877	1013	983	2113	2423	41
859	2837	677	1277	1039	5431	5087	383	1193
6131	599	347	1069	2207	887	23	1453	6067
1459	5647	227	809	353	5711	1997	1301	1279
2633	47	1693	613	6691	443	1019	563	5081
179	1607	5297	2843	257	1063	1249	5437	857

Fig. 28.

ranged. Fig. 26 shows an irregular series, and Fig. 27 shows a pattern for the subsidiary squares, one being a reflection of the other. A resulting square is shown in Fig. 28.

It may also be noted that, from Fig. 26, sets of numbers may be chosen for squares of the 4th, 5th, 7th, and 8th orders, and the possible number of distinct squares which may be formed from these depends on the laws of *permutations* and *combinations*.

TWIN SQUARES.

Two magic squares having like summations and having no numbers in common have been termed "twin squares."

1	599	701	337
281	757	421	179
547	71	389	631
809	211	127	491

29	619	677	313
257	733	449	199
523	47	409	659
829	239	103	467

Fig. 29.

To construct twin squares of the 4th order, we select a table of numbers having 8 or more columns and 4 numbers per column. The tabular differences may be irregular. From the top line of numbers, select two groups of four numbers each, that have like summations. Each of these groups will indicate the columns of numbers to be used in the respective squares. Each square is constructed as was explained in reference to Figs. 4, 5 and 6.

Twin squares of the 4th and 5th orders are shown in Figs. 29 and 30 respectively. In both examples, the above method was employed.

The foregoing methods of constructing prime number squares, by the use of tabular series, are obviously of little use for con-

389	643	6113	5927	1321
5717	2161	179	433	5903
223	5693	5507	1951	1019
1741	809	1063	5483	5297
6323	5087	1531	599	853

2909	3583	653	1097	6151
887	6991	2699	3373	443
3163	233	677	6781	3539
6571	3329	4003	23	467
863	257	6361	3119	3793

Fig. 30.

structing squares of the higher orders, due to the increased difficulty in finding series of the necessary requirements. Large squares have been constructed, however, by other methods.[4]

[4] See squares by Messrs. Shuldham and Muncey in *The Monist*, Oct. 1913, Vol. XXIII, No. 4, pp. 623 to 630.

UMI

PRIME NUMBER TABLES.

To facilitate the finding of sets of prime numbers to be used in the construction of prime number magic squares, the prime number series has been arranged in tabular form in various ways.

One form of table is shown in Fig. 31, which is composed of a lattice having five cells in each line, the columnar length of the table depending on the range of numbers to be tabulated. The cells are counted by odd numbers in natural order from left to right, first through the top line, then through the second, and so on down through the table, a dot being placed in each cell where a prime number falls. In some cases, the table is made more convenient by extending it to form extra columns containing numbers which are duplicates of those in the table proper. Fig. 31 shows two supplementary columns which contain duplicates of the numbers in the first two columns. It should also be noted that the numbers in the supplementary columns must be one cell nearer the head of the table than the numbers they duplicate.

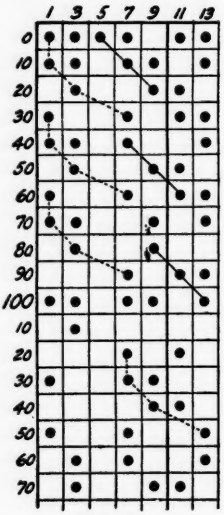


Fig. 31.

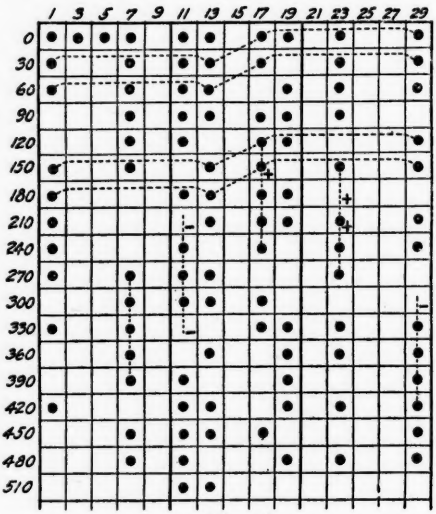


Fig. 32.

At the top of the table, the columns are numbered 1, 3, 5,...., and at the side, the lines are numbered 0, 10, 20, 30,...., the latter numeration depending on the number of cells in a line. The value of a number represented by any dot, may be determined by adding

together the index numbers at the end of the line and column in which the dot lies.

Fig. 32 shows another table in which the increments between lines are 30; and a portion of a still larger form of table is shown in Fig. 33, where the increments between lines are 210.

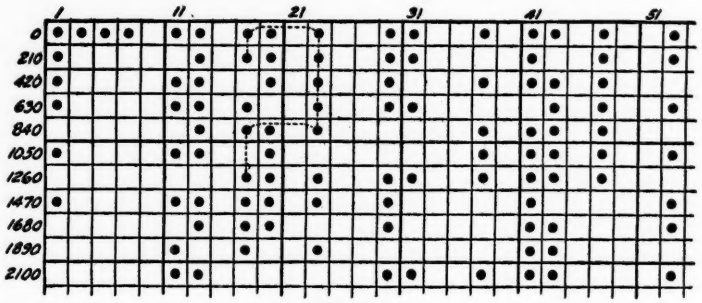


Fig. 33.

To get the best formation of tables, the increments between lines should be a multiple of two or more different small prime factors. In Fig. 31, the increment is $10=2\times5$; in Fig. 32, the increment is $30=2\times3\times5$; and in Fig. 33, the increment is $210=2\times3\times5\times7$. The value of this may be observed by the resulting linear grouping of numbers in the tables.

To illustrate the use of these tables, we will point out a few of the groups of numbers which have been used in the preceding magic squares.

In Fig. 31 will be found three triads, the dots in each triad being indicated by a straight line connecting them. This group of 9 numbers is shown in Fig. 1 and is arranged in magic square formation in Fig. 3. It will be noted that the 9 dots in the table form a symmetrical figure, which indicates an *associated* group of numbers. In these forms of tables, all *associated* groups of numbers will show symmetrical figures, unless the grouping runs over the edge of the table. If the table in Fig. 31 had not been extended, the 9-number group would have appeared irregular if kept within the table.

In the same table will be found four irregular groups having four dots each, as indicated by dotted lines. Each group has the same form, though their relation is unsymmetrical. This group of 16 numbers was used to form the magic square shown in Fig. 6.

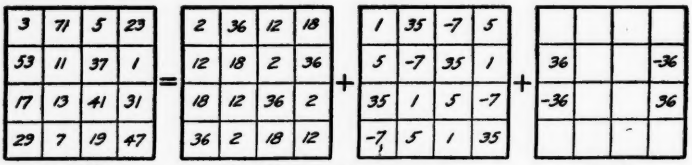
A symmetrical group of 16 numbers is pointed out in Fig. 32, which was used to form the associated square in Fig. 8 and the pandiagonal square in Figs. 9 and 11.

In Fig. 33 is shown an irregular group of 9 numbers. Nine groups of this form were used in constructing the square shown in Fig. 28.

The foregoing rules of construction will aid in the simple formation of magic squares with prime numbers, but these rules are apparently inadequate in certain instances, for example, to construct a pandiagonal 9th order square, or to form a magic cube with prime numbers. The writer believes that these and other problems can be mastered if we bring further irregularities into the magic square series. In the following pages will be shown some of these irregularities, with an introduction of the "kink."

THE KINK.

The "kink" was first discovered in analyzing the prime number square, S = 102. Fig. 34 shows this square resolved into two La



S = 102 — La Hireian Subsidiaries. — Kink.

Fig. 34.

Hireian subsidiaries of Jaina formation and a kink. It will be noticed that this kink involves four cells, rectangular in position, and of equal numeral values, two being plus and two minus. For

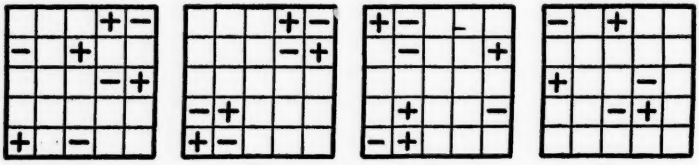


Fig. 35.

convenience of designation we will give this form the term *rectangular kink*. It will be observed in the rectangular kink, that any two values in a horizontal or vertical line, counteract each other; there-

fore, a rectangular kink of any plus or minus values, may be added to any magic square without affecting the magic summations, providing any part of the kink does not fall in a magic diagonal. A rectangular kink may affect a diagonal, providing a second kink is added to correct the fault. Fig. 35 illustrates a few double rectangular kinks by which the main diagonals of the square have been corrected. In the central cell of the square "D," two kink values are neutralized, and are therefore not shown.

Two like rectangular kinks may be combined so as to form a kink that will not destroy the values of any of the magic summations of a pandiagonal square. This form might be termed *pan-*

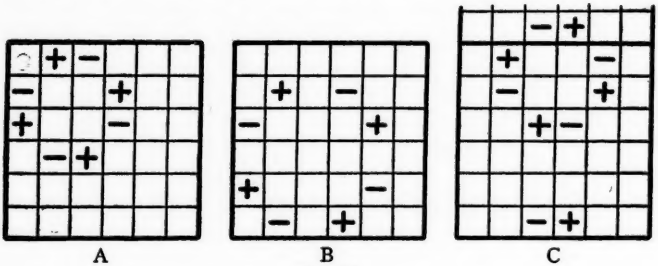


Fig. 36.

diagonal or *octagonal kink*. Fig. 36 shows a few of the latter type, the square "C" showing how the octagonal form may be apparently missing in some cases, due to the kink running over the edge of the square.

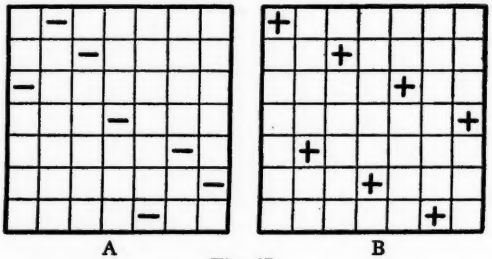


Fig. 37.

minus or plus as shown in Fig. 37. In square "A" the same value is taken from each line, column, and the two diagonals, which

There are also other kinks where all the values are either would mean a lowering of the summations but would not destroy

its principal magic qualities. In square "B" each line, column and all diagonals are affected alike, and may therefore be classed as a *pandiagonal kink.* The values in this latter kink are shown to run in a knight's path, which formation has suggested the term *path kink.*

When applying kinks to squares, it is not necessary that the same numeral values are applied to all the kinks involved. This

	$+n$			$-t$	$+t$	
			$+t$	$+n$		$-t$
$+n$	$-s$		$-t$		$+s$	$+t$
			$+n$	$+t$	$-t$	
	$+s$				$-s$	$+n$
		$+n$				
					$+n$	

Fig. 38.

10	18	1	14	22
11	24	7 +	20	3 −
17	5 −	13	21	9 +
23	6	19	2	15
4	12 +	25 −	8	16

Fig. 39.

1	2	3 −	4	5 −
6	7 +	8	9 +	10
11	12 +	13	14	15
16	17	18	19	20
21	22	23	24	25 −

Fig. 40.

is illustrated in Fig. 38, where are shown a rectangular, an octagonal, and a path kink. It can be realized from this illustration that a magic square series can be greatly distorted by the addition of a few kinks.

The following example will suffice to show the constructive application of the kinks. Fig. 39 is a magic square containing the

	w	x	y	z	
a	277	179	329 −	137	221 −
b	307	203 +	359	167 +	251
c	337	233 +	389	197	281
d	367	263	419	227	311
	397	293	449	257	341 −

Fig. 41.

277	173	179	137	71
307	353	359	317	251
337	383	389	197	281
367	263	419	227	311
397	293	449	257	191

Fig. 42.

251	419	277	197	293
337	257	353	311	179
263	71	389	397	317
449	307	227	173	281
137	383	191	359	367

Fig. 43.

series 1, 2, 3,.... n^2, constructed with subsidiaries of the pattern shown in Fig. 13. The numbers of this square are shown in tabular form in Fig. 40. The position of two arbitrary rectangular kinks has been designated in Fig. 39, and these kinks have been transferred to the same respective numbers in Fig. 40. Any set of numbers with tabular differences of a, b, c, d, and w, x, y, z, that is affected by kinks, as shown in Fig. 40, is susceptible to transformation into a

magic square. In Fig. 32 are shown by vertical dotted lines, five sets of numbers which have been tabulated in Fig. 41, the strings of the former being arranged as columns in the latter. The selections in Fig. 32 were made with the kinks in mind, which are indicated with small plus or minus signs. Where the minus signs occur, we have substituted the number five cells above in the same column, and where the plus signs occur, the number five cells below has been used; these differences of position being equal to 150, which is the numeral value of the kink used. This kink value has been added to the cells indicated in Fig. 41 which gives the numbers shown in Fig. 42. The numbers in Fig. 42 are now transposed into the magic square Fig. 43, in the same respective order that the numbers of Fig. 40 are shown in Fig. 39.

It can be seen from the foregoing that unusual irregular series, formed by the use of kinks, may be anticipated, and in a great many cases, found and arranged in magic squares. The writer believes that the 3d order cube will be solved in this manner, which will be referred to in the following pages.

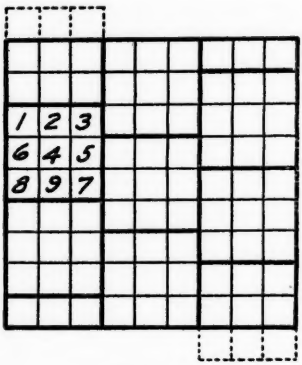


Fig. 44.

677	1069	601	761	269	401	11	577	887
937	313	139	151	487	389	643	1181	1013
307	101	59	467	1289	1399	1117	281	233
239	351	371	1523	499	37	17	43	173
1061	811	569	23	83	157	509	669	1371
71	31	103	53	787	1051	1547	883	727
431	773	1283	1151	379	439	443	293	61
1109	191	557	461	541	397	359	977	661
421	613	571	663	919	983	607	349	127

Fig. 45.

Another method of using the kink, is to construct the desired magic using as few composite numbers as possible, and then add various kinks at the places where the composite numbers occur; the process being continued until all the composite numbers are eliminated. The pandiagonal square of the 9th order will undoubtedly be accomplished in this way.

For this square, the preliminary series (containing as few

composites as convenient) should be so chosen as to allow each of the two subsidiary groups of numbers to be divided into three triads of equal values. The numbers are arranged in quarrels in the subsidiary squares, as shown in the pattern, Fig. 44, the triads being placed in vertical strings. The subsidiary squares are similar in pattern and are placed 90 degrees apart, or, one subsidiary may be reflected on either of its two diagonals.

Fig. 45 illustrates a pandiagonal square constructed by Chas. D. Shuldham. By the above method he has succeeded in transforming it to contain as low as six composites, which are indicated by circles.

THE CUBE.

To the writer's knowledge, the prime number cube of the 3d order has not yet been constructed. The kink will undoubtedly aid in its construction, and the following example will suffice to show the application of the kink to the cube.

Fig. 46 shows the three respective layers of one of the various magic cubes constructed with the natural series of numbers, and

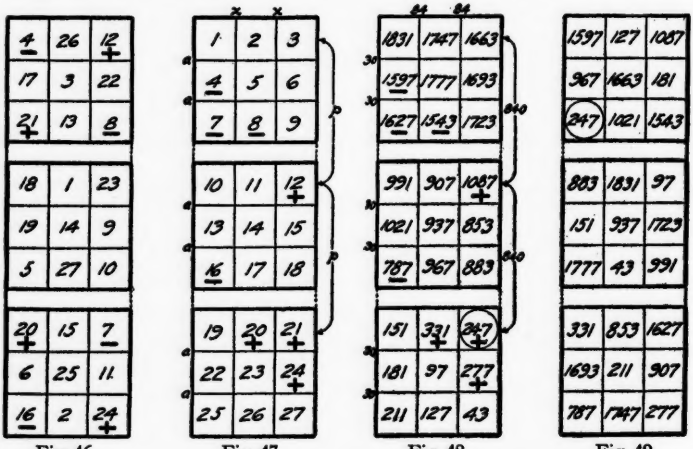


Fig. 46. Fig. 47. Fig. 48. Fig. 49.

Fig. 47 shows this series in tabular form. A magic cube of ordinary construction must have a series whose tabular differences are a, a and p, p and x, x, as indicated in Fig. 47, the numbers being arranged in the cube in the same order as any cube of a straight series.

The simple *cubic kink* is shown diagrammatically in Fig. 50 and has the form of a right parallelopiped. A single kink of this form can be added to the 3d order cube in only one way, that is, its eight values must fall in the eight corner cells of the cube, otherwise the summation of the diagonals would be altered.

The position of a simple kink is indicated in Fig. 46, and is transposed to the same respective numbers in Fig. 47.

The series of prime numbers for the cube would be discovered in the same manner as were the numbers in Figs. 42 and 43, that is, find any set of numbers in the prime number table that has tabular differences, as indicated by letters in Fig. 47, disregarding any composite numbers that may occur where the kink is indicated. For the plus values of the kink, a new set of values is discovered in the table which has the same geometric relation to each other as the originals. This is diagrammatically illustrated in Fig. 51. Care should be taken that a new set for the minus values can also be found in a symmetrically opposite direction.

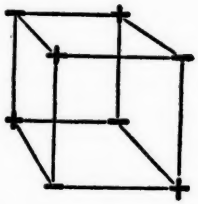


Fig. 50.

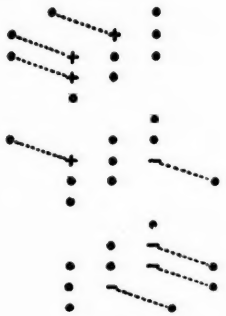


Fig. 51.

According to the above method, the writer has succeeded in finding, in a table of the form shown in Fig. 32, the series shown in Fig. 48, which contains the one composite number indicated with a circle. These numbers being transposed into a cube according to Figs. 46 and 47, produces the magic cube shown in Fig. 49.

Combinations of kinks can be added to cubes in various ways which the reader can easily discover for himself. The variations in tabular differences and in kink formation and combinations would apparently indicate that the discovery of a prime number series for a cube is possible. There is greater freedom in the

application of kinks to the 4th order cube, though the writer has not investigated beyond the 3d order.

It also seems possible that the cube of the 4th order may be constructed by an extension of the *method of pseudo-complementaries.*[5]

Patience and perseverance will be found to be the principal requirements in solving these difficult problems in prime number magics. Who will claim the honor of being the first to solve them?

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BOOK REVIEWS.

THE ELEMENTARY FORMS OF THE RELIGIOUS LIFE. A study in religious sociology. By *Emile Durkheim,* professor of the faculty of letters at the University of Paris. Translated by *J. W. Swain, M.A.* London, George Allen & Unwin, 1916. Price 15s. net.

This book must be valued from three different points of view. It contains reinterpretations of the principal social phenomena of primitive peoples; it contains a theory of the genesis of knowledge with doubtful philosophic implications; and it contains what we may assume for the present to be M. Durkheim's definitive pronouncement on the nature and the future of religion. All of these strands of argument are bound together by M. Durkheim's well-known theory of the group-consciousness, but this theory itself must be assigned different values in these three developments.

It is in the more purely anthropological aspect that this book is most successful. Here M. Durkheim's views must be judged in comparison with those of the older interpreters such as Tylor, Müller, Lang, Frazer, Jevons, Robertson Smith, Mannhardt. As in most works of the sort, the author is most convincing when he sticks closest to the facts, when he is least metaphysical, and when he is engaged in refuting his predecessors. In fact, he is most convincing when he is showing us what the phenomena of primitive religion do *not* mean. M. Durkheim confines his observation almost entirely to Australia, and his theory of Australian totemism is distinctly the best that has yet been evolved. Why? Because he is able to show that totemism is not animal worship, that it is not derivative from ancestor worship, or from the "nature cult"; the totem is not a name; the group totem is not, as Frazer holds, a development from the conception totem. M. Durkheim's theory is the best because it is the nearest to being no theory at all. And when he comes

[5] See "Even Order Magic Squares with Prime Numbers," *The Monist,* Jan. 1916, Vol. XXVI, No. 1.

to state it in positive terms, he finds almost as much difficulty as his predecessors in avoiding intellectualization. His "group-consciousness" is a contribution. But is it capable of articulate expression? "The totem," he says, "is the flag of the clan." This is just what some of the earlier theorists have said. It is a "collective representation." It has for the group-consciousness a significance quite different from the significance which that animal or plant has for the individual consciousness. We are not sure that this means anything more than that it is incapable of explanation. Totem is the origin of the idea of force. "Religious force is nothing other than the collective and anonymous force of the clan....If religious force, in so far as it is conceived as incorporated in the totemic emblem, appears to be outside of the individuals and to be endowed with a sort of transcendence over them, it, like the clan of which it is the symbol, can be realized only in and through them; in this sense, it is imminent in them and they necessarily represent it as such." M. Durkheim has given reason to believe that the examination of the individual consciousness is inadequate to explain social phenomena. He does not convince us that his social psychology is anything but an admission of the inexplicable, that the "group-consciousness" and the "collective representation" are more than a definition of the limits of individual psychology.

We should have liked to discuss the theory of the "origin of the categories" at length; although the exposition of this theory is much slighter than its place in the analytical table of contents would lead us to expect. It is open to the same charge of negativity, and leaves epistemology, we think, precisely where it was before. The theory of the nature of religion is stated in the conclusion. We have only space to draw attention to one difficulty. On page 416 we read that "the real function of religion is not to make us think, to enrich our knowledge, nor to add to the conceptions which we owe to science others of another origin and another character, but....to make us act." On page 428 we find that a religion "is not merely a system of practices, but also a system of ideas whose object is to explain the world."

The whole book is intensely interesting. The translation is good, but of less literary finish than the original. η

ELEMENTS OF FOLK PSYCHOLOGY. Outlines of a psychological history of the development of mankind. By *Wilhelm Wundt.* Authorized translation by *E. L. Schaub, Ph.D.,* Professor of Philosophy in Northwestern University. New York: Macmillan. Price 15s. net.

Durkheim's *Formes élémentaires de la vie religieuse* and Wundt's *Elemente der Völkerpsychologie* appeared in the same year, 1912. In the preface to the present work Professor Wundt states the difference in method between the earlier *Völkerpsychologie* and this shorter book. "Instead of considering successively the main forms of expression of the folk mind, the present work studies the phenomena, so far as possible synchronously, exhibiting their common conditions and their reciprocal relations....The chief purpose of investigations in folk psychology must be found in a synthetic survey." This is in accord with Durkheim's theory that the religious phenomena must not be isolated by the investigator from the rest of the social life of a people. Otherwise the books of the two men are strikingly different. Durkheim's psychology

is metaphysics. Wundt's psychology is descriptive anthropology. His method is descriptive and historical. He divides the stages of culture into four: (1) primitive man, including prehistoric man and such existing tribes as the Veddahs, Bushmen and Negritos; (2) the totemic stage, including the Australians and the Iroquois; (3) the "age of gods and heroes," the age of the folk epic; (4) the "development to humanity," which includes a discussion of "world-empires" and "world-religions." In each of these stages he takes up cult, social organization, myth, art, language; except that in the last stage the treatment is vaguer and these divisions are abandoned.

In his account of primitive and savage society Wundt is in general sound, but unsatisfying. When we turn to totemism, for example, he gives the impression of painstaking common sense. He is certainly right in rejecting the "eugenic" theory of exogamy, and in combating the "conceptional" theory of the totem. But it is improbable that the group totem is (as Wundt apparently holds) an outgrowth of the individual totem. Wundt is an animist. "Totemic ideas arise as a result of the diremption of primitive soul ideas into the *corporeal soul* and the *breath-* and *shadow-soul*" (p. 192). The soul is regarded "as a moving form, particularly as an animal, a bird, a rapidly gliding snake, or a lizard." We are inclined to believe that this "breath-soul" which totemism introduces was at first, and in fully developed totemism, much more indefinite and impersonal than Wundt would lead us to suppose. And he does not succeed in showing the relation between totemism as a social organization and totemism as a religious cult.

For the rest, Wundt is less concerned with explaining motive and meaning than with explaining the development of forms. Thus, his account of art is taken up largely with the development of the stringed instrument out of the bow, and kindred problems; he engages in a discussion of the beginnings of domestication of animals. The major part of his subject matter, in short, is not psychological at all; it belongs, in the earlier stages, to descriptive anthropology, and in the later stages, to the philosophy of history. And of the role which the sexual instinct plays in the religion and mythology of primitive peoples (indeed in all religion) Wundt has almost nothing to say. The psychoanalysis of myths, pursued by some of Freud's disciples, is surely capable of throwing considerable light on the primitive mind. It is possible that Wundt is still under the domination of a Hegelian conception of history. Although he criticises Hegel for applying a "logical schematism which is in large measure imposed upon history," his own account is very rationalistic. The book is a sound and valuable handbook, enriched by Wundt's ideas. But we think that any further advance in folk psychology is conditioned by advance in individual psychology.